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ABSTRACT

Presented is a description of a mathematics program for kindergarten through grade six, as recommended by the State Education Department of New York. The curriculum for each grade in elementary school mathematics, in this publication, is organized into five major strands: (1) number and numeration; (2) operations with whole numbers; (3) operations with fractions; (4) probability and statistics; and (5) geometry and measurement. The guide states that the strands should be viewed as intricately related, and that teachers should use their own judgment concerning the order of presentation of the topics. The document gives many examples of problems and activities that can be used to help communicate the listed objectives to pupils. (MP)

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Mathematics K–6

A Recommended Program for Elementary Schools

ED199041

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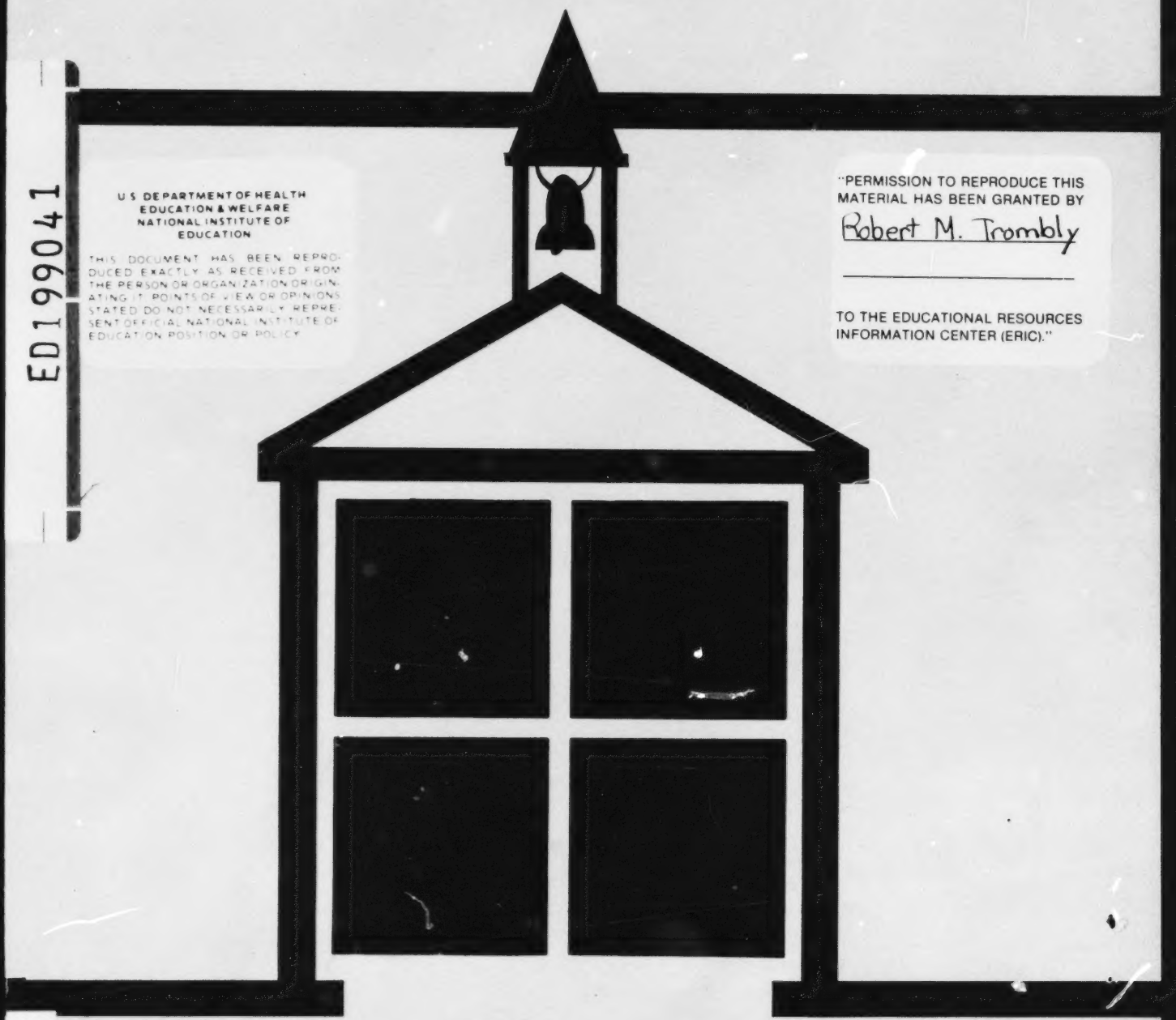

SE 034 058

The University of the State of New York / THE STATE EDUCATION DEPARTMENT
Bureau of General Education Curriculum Development
Albany, New York 12234
1980

The course of study for the first eight years of full time public day schools shall provide for instruction in at least the twelve common school branches of ARITHMETIC, reading, spelling, writing, the English language, geography, United States history, civics, hygiene, physical training, the history of New York State and science.

EDUCATION LAW — *Section 3204, Item 3*

Foreword

The general form and character of the mathematics program described in this publication were determined by an advisory committee consisting of:

Leroy Callahan, State University of New York at Buffalo
Edward Cunningham, Roslyn Union Free School District
Mildred Dominy, State University College at Plattsburgh
Blanche Gladstone, New York City Public Schools
William Schwarz, East Greenbush Central School
Sister Colletta Woods, Buffalo Diocesan Schools

The manuscript for the syllabus was written by Elaine Mintz, supervisor of mathematics at Plainedge Union Free School, and Edward Cunningham, teacher at Harbor Hill Elementary School in Greenvale, New York. During the years that followed, the material was discussed with and reviewed by hundreds of mathematics teachers; revised; updated; and, finally, field tested in elementary school classrooms in more than 40 districts throughout the State. After the field test, the material was again revised — this time in accordance with the suggestions of the teachers who had actually used it. The final edition was written by Kristina Leeb-Lundberg, an associate professor in the Department of Elementary Education at City College of New York, with contributions from George Aman, principal of the Chestnut Hill Elementary School in Liverpool, New York.

The development of the syllabus began under the direction of Frank Hawthorne, chief of the Bureau of Mathematics Education (now retired), and was continued by his successor, Fredric Paul. John J. Sullivan, Associate in Mathematics Education, held specialist responsibility for the project; and Peter Martin, Associate in Elementary Curriculum (now retired), coordinated the writing and review activities. Rita A. Sator, Carolyn Herkenham, and Donna McCarthy of the Bureau of General Education Curriculum Development prepared the final manuscript for publication. The cover was designed and executed under the direction of Fred Haber, supervisor of the Audiovisual Center, and the illustrations were done by Lesa Schissler.

The contributions of all who helped to shape *Mathematics K-6* are gratefully acknowledged.

SOUP
metric length and area

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If we do not know to which port we are sailing, no wind is favorable.
— Seneca

Introduction

The curriculum for elementary school mathematics described in this publication is organized into five major strands:

Number and numeration
Operations with whole numbers
Operations with fractions
Probability and statistics
Geometry and measurement

Although the writing of a syllabus requires the placement of topics in some particular sequence, teachers should use their own judgment concerning the order of presentation of topics. The strands should be viewed as intricately related, with each topic supporting and illuminating many other topics. In adapting this recommended curriculum for use in local school situations, the following items should be considered:

. *Articulation*

The best lessons are those which integrate topics from several strands in meaningful ways.

. *Problem Solving*

A major reason for the high priority given to mathematics in school programs is its great usefulness in solving problems. Every lesson should have elements which help students to develop problem-solving skills. Two publications related to this aspect of study, *Problem Solving* and *Creative Problem Solving*, have been developed by the State Education Department and are available upon request from the Bureau of General Education Curriculum Development and from the Bureau of Mathematics Education.

. *Terminology*

Terminology is less important than concept development, but it is important. Some of the technical words in the syllabus will be difficult for children. Teachers should use the terms correctly, but be patient with their students' struggles to master them.

- *Measurement*

  Children should learn to measure carefully by actually measuring things—to some purpose, if possible. In primary school, children should estimate measures, compare quantities, and measure objects using nonstandard units (pencils, rods, handspans, ...) before using either customary or metric units of measure.

  This syllabus recommends that the metric system be learned, with only cursory attention given to the customary system of measure (inches, pounds, quarts, ...).

- *Fractions*

  Computation with common fractions should receive less emphasis than in the past. In concept development, common fractions remain eminently useful as, for example, in understanding probability. Decimal fractions, however, are increasingly useful; and this syllabus introduces them earlier than was common in the past.

- *Calculators*

  One of the reasons for the increased use of decimals is the prevalence of calculators. Soon every student will have access to a calculator. It is advisable to introduce children to these instruments which are increasingly important in our lives. Teachers will be able to find many valuable and exciting ways to use calculators to enrich instruction.

- *Algorithms*

  A goal of school mathematics is to help children to become proficient with mathematical algorithms. Although algorithms are mechanical procedures for the most part, it is important for children to understand the basic processes involved. For this reason, many teachers use intermediate algorithms *for a short time* to help children understand standard algorithms.

- *Understanding Mathematics*

  A fundamental assumption underlying these curriculum recommendations is that mathematics must be understood in order to be lastingly useful. Rules and algorithms learned by rote may be remembered for a while, but will eventually be forgotten. An overemphasis on memorization in the teaching of mathematics could result in confusion, resentment, and a lasting antipathy for mathematics. The recommendations in this publication are designed to support teaching for understanding.

The Curriculum

Kindergarten

KINDERGARTEN — **NUMBER AND NUMERATION**

Develop an awareness of the concepts, words, and symbols related to numbers as used in daily living.

Sorting
Classifying

Prepare for number concept development through sorting and classifying activities, using concrete objects such as buttons, blocks, and bottle tops. Explore likenesses and differences (color, shape, size, ...). Compare sets with *more*, *less*, and *the same* amounts.

Sets

Use the term *set* synonymously with terms such as *group*, *lot*, *pile*, *bunch*, *heap*, ...

Equality, inequality

Extend quantitative comparisons in many different situations to terms such as *bigger than*, *greater than*, *same size as*, *less than*, *equal to*, ...

One-to-one correspondence

Develop the idea that if two sets can be matched or put into one-to-one correspondence, then they have the same cardinal number.

Counting
Cardinal numbers, 1-10

Develop the idea that the last number counted in a set tells how many things there are in a set (*cardinal number*). Do this gradually, first for the numbers 1 - 5, then for the numbers 1 - 10.

Explore, through counting, the cardinal numbers of a multitude of sets and collections of real objects, such as the number of boys and girls, cookies, milk cartons, toy trucks, mittens, ...

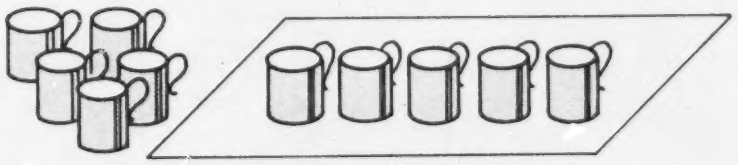
Integration

Integrate the comparison of sets and counting with other activities such as cooking, housekeeping, stories, games, and blockbuilding.

Conservation

Engage in numerous individual manipulative experiences to perceive that the cardinal number of a set remains the same no matter how the elements are arranged.

Example:



Order	Order sets of objects (dolls, dishes, blocks, ...) from smallest to largest, and from largest to smallest.
	Develop the concept of *first*, *last*, and *middle*.
	Use ordinal number names from *first* through *tenth*.
Numerals	Observe *numerals* (names for numbers) in the environment.
	Proceed gradually from concrete manipulation to pictorial and symbolic representation.
	Identify number names orally through 15.
	Develop gradually the ability to read and write numerals from 0 - 10.
Number line	Use a floor number line for counting forward and backward.
	Place sets of objects in order so that they correspond to a number line.
	Compare the cardinal number of a set of concrete objects with its representation on a number line.

KINDERGARTEN — OPERATIONS WITH WHOLE NUMBERS

Combining	Put two simple sets together to produce a new set, the cardinal number of which is less than 10.
Sharing	Share sets of objects (cookies, toys, crayons, ...).
Sequence	Explore the idea of *one more* through sets of objects, and through a sequence of sets containing 1 - 10 objects.
	Discuss and use the concepts *before*, *after*, and *between*.
Real-life problems	Investigate various numerical problems that arise in school, such as attendance, milk orders, and bus loadings.

KINDERGARTEN — OPERATIONS WITH FRACTIONS

	Develop an awareness of fractions in daily use.
Inequality	Discuss such ideas as "A whole is *more than* a half." and "A half is *less than* a whole."
Two equal parts of a whole	Explore the fraction concept, using halves.
	Fold various geometric shapes (circles, squares, rectangles, ...) into halves.
Many equal parts of a whole	Participate in sharing experiences which show that a whole (such as a cake, an apple, or an orange) may be divided into many equal parts.
Equal parts of a set	Participate in sharing experiences which show that sets of objects (such as a bag of candy or a set of blocks) can be divided into equal parts, beginning with halves and observing that the total number of objects is unchanged.

Examples:

Color $\frac{1}{2}$ of a picture of an object.

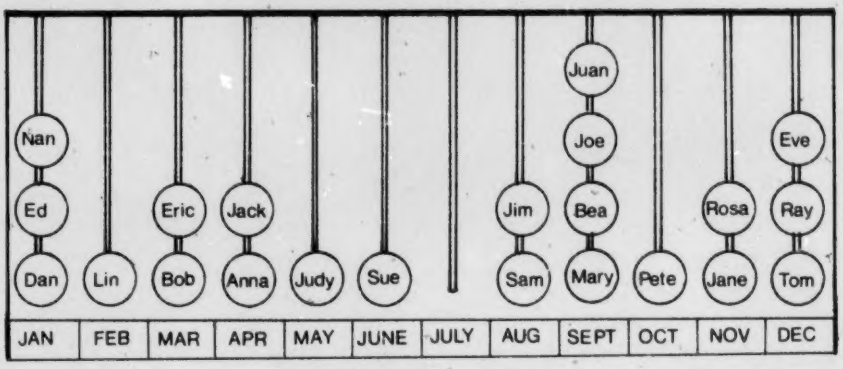
Color 1 out of 3 objects of the same type.

KINDERGARTEN

PROBABILITY AND STATISTICS

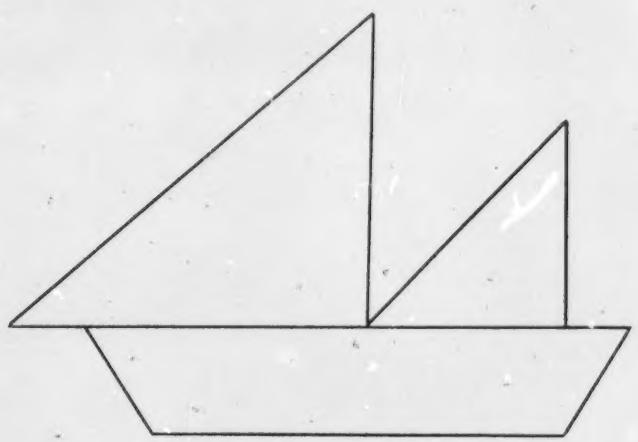
Classifying information	Participate in sorting and classifying activities with blocks, toys, cookies, ..., observing likenesses and differences. Use two categories at a time. Line up with other children according to gender, eye color, hair color, ...
Equality, inequality	Compare materials in terms of *more*, *less*, and *the same amount*.
Concrete graphs	Pile blocks in two stacks to represent children who prefer apple or grape juice, dolls or trucks, ...
Chance events	Use colored spinners for decision making in games and for choosing activities. Discuss the certainty and uncertainty of events, such as whether a beanbag will land on number 3, whether a blindfolded child will pick a blue block from a box, whether a dog will fly, ...
Estimation	Participate in activities which involve anticipating outcomes, such as stacking blocks until the pile falls and then guessing (*estimating*) at what height other piles will fall.
Language	Use the language of chance, such as it is *more*, or *less*, *likely* that Henry (or Susan, or Joe, or ...) will be in school tomorrow.
Graphs	Discuss graphs such as the one below.

MONTHS IN WHICH WE WERE BORN



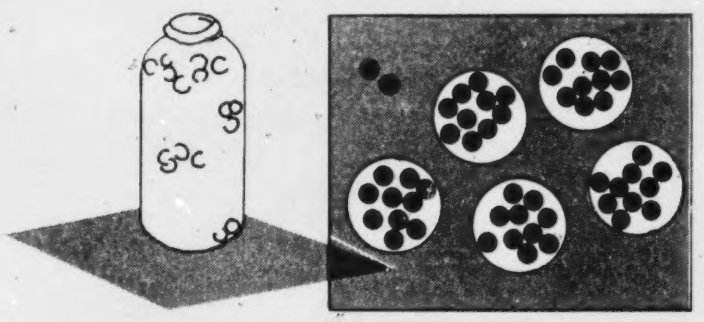
Guessing	Play guessing games.

Comparisons Length, height, width, depth Equality, inequality Distance	Compare the length, height, width, and depth of various objects such as ribbons, toys, blocks, ... and of other children (e.g., their hands and feet). Use terms like *longer than, taller than, smaller than, shorter than, as long as, ...* . Then compare distances, using such terms as *farther* and *nearer*.
Estimation	Practice estimating (guessing) sizes, using phrases like *about as long as, almost as long as, wider than, ...* .
Mass	Participate in activities that involve weighing, first in the hands and then on balance scales, using such terms as *heavier* or *lighter than* and *weighs more, less,* or *the same as ...* .
Capacity	Use sand, water, ... to compare the capacity of containers ("The pail, cup, ... holds *more, less,* or *the same as ...* .").
Temperature	Compare (weather, food, ... temperatures) in general terms. For example, "Today is *warmer than* (*cooler than, hotter than, as cold as*) yesterday."
Time	Use an egg timer, an hourglass, ... to compare the duration of eating time, story time, ... (e.g., takes *longer, less time than,* or *as long as*).
Nonstandard units of measure	Explore various nonstandard units of measure, such as blocks, books, and children's feet, handspans, bodies, ..., for length and distance; buttons, blocks, bottle tops, ..., for weight; cupfuls, bowlfuls, ..., for capacity; and faucet drips, ..., for time.
Money	Use real money for activities such as shopping, to learn the names of bills and coins.
Shape	Explore the environment and note objects with geometric shapes (windows, doors, pictures, signs, tiles, tables, ...).
Counting	Collect boxes, bricks, and other three-dimensional shapes. Relate counting activities to geometric activities, such as finding out how many faces, edges, corners, ... a cracker box has.
Patterns	Engage in pattern-forming activities with blocks and puzzles. Create geometric pictures and designs from cut-out shapes. Example:



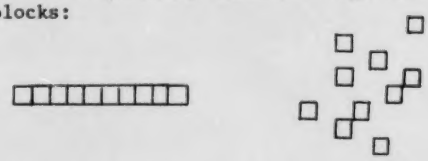
First Grade

FIRST GRADE	NUMBER AND NUMERATION
Number concepts Sets	Continue to develop number concepts through sorting and classifying activities, using concrete objects. Compare sets with *more*, *less*, and *the same* amounts. Use real-life situations, stories, games, art and social studies projects, ... to reinforce familiarity with number symbols and words. Review cardinal numbers to 15, matching concrete sets one-to-one when needed. Match number words and symbols in working with sets of objects from zero to 15.
Counting	Count forward and backward by ones and twos on a number line. Develop the ability to give the correct cardinal number of a set of two, three, or four objects, without counting.
Number line	Using number lines and number charts, . Count to 100. . Name the whole number immediately before and immediately after any whole number from 1 - 99. . Name the whole number between any two numbers from zero to 100 that are two apart, such as 65 and 67.
Order	Use ordinal numbers up to 10 with sets of objects and the number line.
Even, odd	Explore the concepts of even and odd numbers, using sets of children and concrete objects. Begin by pairing.
Grouping	Participate in activities with concrete objects, laying the groundwork for an understanding of the grouping concept.
Estimation	Use manipulative materials (beans, popsicle sticks, peanuts, ...) for estimating quantity and for grouping into sets of tens and ones.



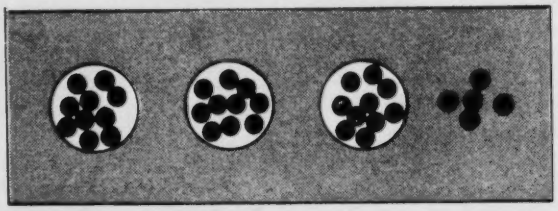
One-to-many correspondence

Consider the idea of *one-to-many correspondence* with a single object representing a group of 10 objects. For example, a dime can be substituted for 10 pennies, and one "long" rod can be substituted for 10 small blocks:



Represent any number from 1 to 100, using sets of counters, bundles of sticks, or metric base 10 blocks.

Example:

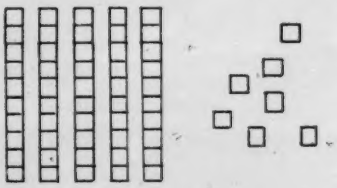


35 = 3 groups of ten and 5 ones

Numerals

Study the meaning of each digit in numerals representing numbers less than 100, using concrete materials.

Example:



57 = 5 tens + 7 ones

Use expanded notation for two-digit numbers.

Example:

35 = 3 tens + 5 ones

Fractions

Recognize fraction numerals with a numerator of 1.

Money notation

Recognize money notation to $9.99.

FIRST GRADE

OPERATIONS WITH WHOLE NUMBERS

From concrete to abstract

Use sets of concrete objects and number lines for quantitative explorations.

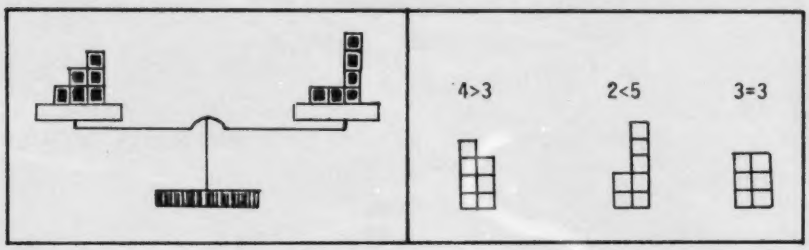
Develop mental and written computational skills as an outgrowth of concrete and problem-solving activities.

Proceed gradually from concrete manipulation to pictorial and symbolic representation.

Equality, inequality

Using concrete objects and balance scales, continue to develop an understanding of terms such as *larger than, smaller than, less than, equivalent to,* ... and the symbols used to represent these relationships.

Examples:



Renaming

Develop the idea that a number has many names.

Example:

5	4 + 1	2 + 1 + 1 + 1
five	3 + 2	6 - 1
cinco	1 + 1 + 3	8 - 3

Addition, subtraction

Practice addition and subtraction facts through 10.

Study remainders when all, some, or none are taken from a set of objects.

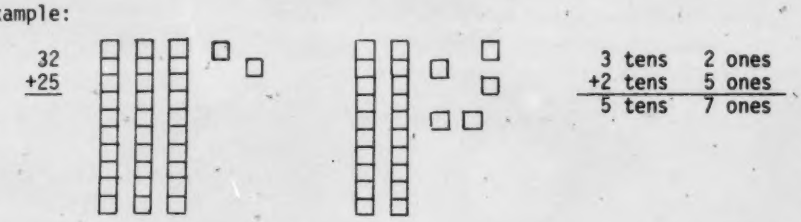
Develop the ability to add three numbers (sum to 10 or less).

Solve addition and subtraction problems in which the sum is less than or equal to 18.

Place value, 2 digits

Add and subtract two two-digit numbers that do not require regrouping, using base 10 materials.

Example:



Operation symbols

Use plus (+), minus (-), and equals (=) signs in horizontal sentences before vertical algorithms:

1 + 2 = 3	1	2	3
2 + 2 = 4	+2	+2	+2
3 + 2 = 5	3	4	5

Read and complete number sentences with place holders such as:

5 + 2 = ☐ 7 - 5 = ☐

5 + 2 < ☐ 7 - 5 > ☐

Inverse operations

Develop the idea that addition and subtraction are "opposite" operations, also called "doing-undoing" operations.

Example:

$2 + 3 = 5 \quad 5 - 2 = 3 \quad 5 - 3 = 2$

Zero

Discover the special role of zero in addition and subtraction.

Example:

$3 + 0 = 3 \quad 3 - 0 = 3$

Commutative law

Discover the basic idea that the *order* of the addends in addition does not change the sum.

Example:

$3 + 2 = 5 \quad 2 + 3 = 5$

Patterns

Discover patterns for sums and differences using

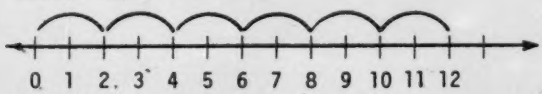
- Concrete materials
- Tables

$1 + 1 = 2$ ●●
$2 + 2 = 4$ ●●●●
$3 + 3 = 6$ ●●●●●●

8	
0	8
1	7
2	6
3	5
4	4

take away 2
10 → 8
9 → 7
8 → 6
7 → 5

- Number lines



add 2

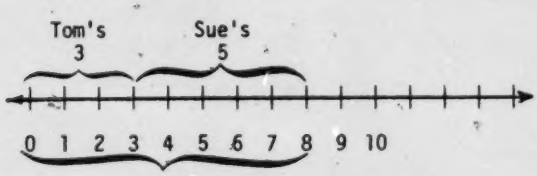
Story problems

Orally explore problem situations in the classroom which have numerical implications.

Write simple number stories.

Use concrete objects, pictorial representations, and number lines to represent and solve story problems.

Example: Tom has three rockets and Sue has five rockets. When they play together with all their rockets, how many do they have?



Translate the story problem into a number sentence.

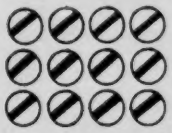
$3 + 5 = 8$

Readiness for multiplication

Arrays

Put concrete objects in rectangular arrays. Use the words *rows* and *columns*.

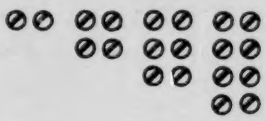
Example:

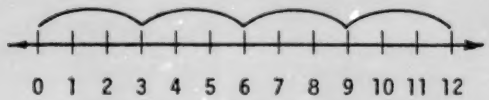


Number line

Explore repeated addition, using arrays and number lines.

Examples:





Readiness for division

Use classroom situations to become familiar with the idea of sharing.

Addition tables

Identify various patterns in addition tables.

Example:

+	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

Use addition tables to find differences as well as sums.

FIRST GRADE

OPERATIONS WITH FRACTIONS

Use real-life situations to develop awareness of fractions in daily use.

Equal parts of of a whole

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$

Develop the idea that a unit fraction represents one of several equal parts of a whole, using concrete objects and number lines.

Develop an understanding of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ by dividing clay, food, and other items into two, three, and four equal parts.

Fold and cut paper circles, squares, and rectangles into halves and fourths. Label the parts.

Verify fractional equivalences by superimposing models of two halves, three thirds, and four fourths on a model of a whole.

Equal parts of a set

Divide heaps of beans, blocks, and similar materials into halves, thirds, fourths, and eighths.

Readiness for ratio

Explore many-to-one correspondence in preparation for the concept of ratio.

Example:

2 shoes for each child
5 fingers for each hand

Story problems

Verbalize experiences such as:

"When Bobby and I shared ten crayons, we each took half. Bobby got five, and I got five."

Money notation

Continue to improve skill in recognizing and interpreting money notation.

FIRST GRADE

PROBABILITY AND STATISTICS

Data gathering

2 categories

Collect, organize, and graph a variety of data relating to preferences ("Who likes apples?" "Who likes bananas?"), clothes ("Who is wearing red today?" "Who is wearing blue?"), ... using two categories at a time.

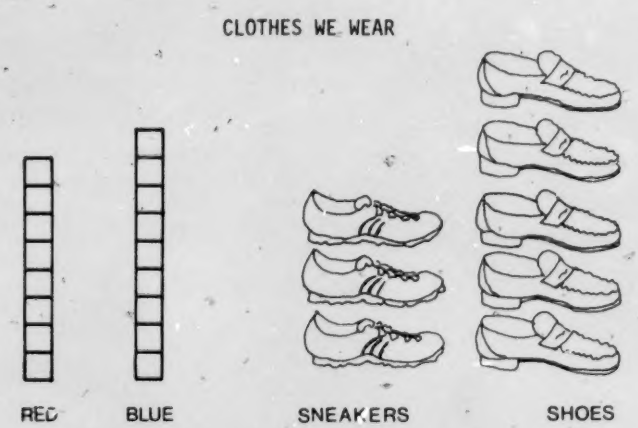
More than 2 categories

Extend information gathering to several categories (e.g., types of leaves collected, pet preferences, favorite colors, beginning letters in names).

Recording

Use blocks and pictographs to record information.

Example:



Equality, inequality

Compare sets first with *more*, *less*, and *the same* amounts, and then with the number of things in each.

Tallying

Practice tallying information in games and classroom activities.

Chance events

Discuss situations which involve the likelihood of events happening, such as "How sure can we be that it will rain this afternoon? ...that our team will win? ...that all cars in the parking lot are blue?"

Toss coins, paper cups, ..., predicting and recording the outcomes.

Arrangements

Arrange a given number of objects in several equal sets, such as 12 objects in four sets, or nine objects in three sets.

Record the answers to questions such as "How many pairs of numbers have a sum of 10?"

10	
0	10
1	9
2	8
3	7
4	6
5	5

0 + 10 = 10
1 + 9 = 10
2 + 8 = 10
3 + 7 = 10
4 + 6 = 10
5 + 5 = 10

Making change

Consider questions about the possible number of ways to make change using pennies, nickels, and dimes.

Example:

What different combinations of coins are equal to a quarter?

Fairness

Play guessing games, taking into consideration the *fairness* of each game.

Example:

The teacher tosses two coins on the table. If both coins turn up heads, the girls win. If both coins turn up tails, the boys win. In other cases, the teacher wins. This game is unfair in favor of the teacher.

FIRST GRADE

GEOMETRY AND MEASUREMENT

Comparison of measurements

Continue concrete activities which involve experiences with equality and inequality while comparing length, distance, mass, capacity, temperature, and time.

Estimation

Make estimating an integral part of measurement activities.

Nonstandard units of measure

Continue to use nonstandard units of measure.

Standard units of measure

Develop an understanding of the need for standard units of measure.

Length
Meter, decimeter, centimeter

Use *meter* and *centimeter* in measurement activities. Introduce *decimeter* when practicing grouping by tens.

Mass
Kilogram

Develop familiarity with the *kilogram* by weighing heavy objects, such as a telephone book, other children, ...

Capacity
Liter

Develop familiarity with the *liter* by pouring water or juice.

Temperature

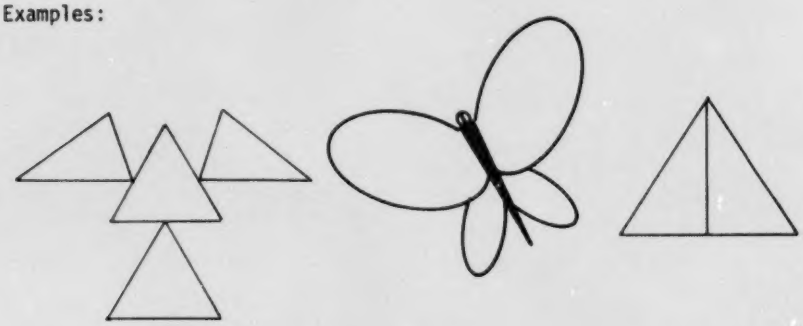
Compare a Celsius thermometer to the number line, and then identify boiling point (100°) and freezing point (0°).

Time

Study time to the hour, day, month, and year. Make use of clocks and calendars.

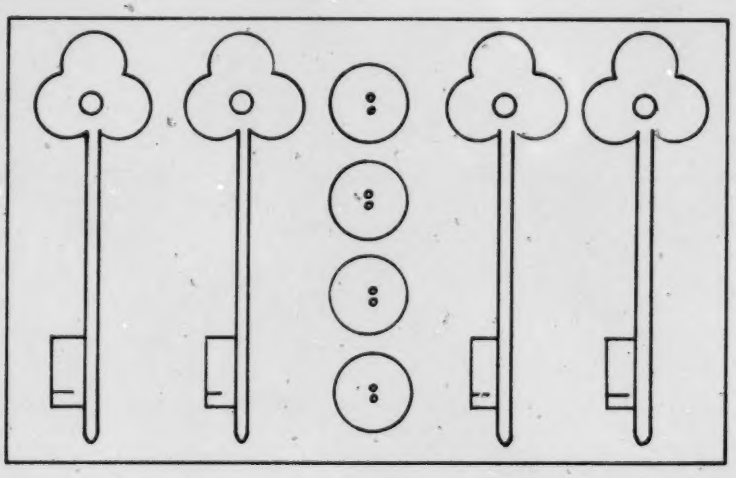
Money	Exchange pennies for nickels and dimes (when grouping by fives and tens).
Customary units of measure	Recognize customary units of measure only as they appear incidentally.
Shape	Observe shapes in the environment (stop signs, railroad crossing signs, windows, desk tops, ...). Study their likenesses and differences.
2 dimensions	Identify circles, squares, rectangles, and triangles used in everyday life (e.g., circular clock faces).
Symmetry	Create symmetrical designs using ink blots, cut-outs, tiles, blocks, and geoboards.

Examples:



Shape fitting	Assemble geometric shapes to make patterns.
3 dimensions	Discover properties of cubes, rectangular prisms, spheres, cylinders, pyramids, ..., while using them in measurement activities.
Position	Place objects in or on enclosures made with strings, or with rubber bands on geoboards. Use the terms *inside* and *outside*.
Patterns	Create patterns with concrete objects (buttons, keys, stamps, ...) and pictures.

Example:



Second Grade

SECOND GRADE **NUMBER AND NUMERATION**

Cardinal numbers

Use cardinal numbers to 1000.

Explore number sequences in the range 1 - 1000, in pattern searches.

Ordinal numbers

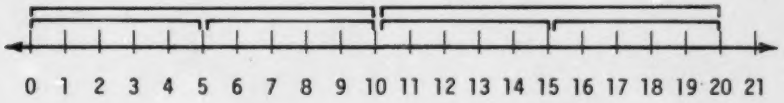
Develop and use ordinal numbers to thirty-first.

Counting patterns

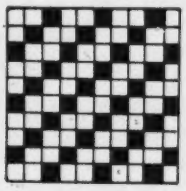
Count by twos, threes, fours, fives, and tens, progressing from concrete objects to number lines and number charts.

Examples:

Count by 5's and 10's on the number line:



Color in 3's on a 100-square:



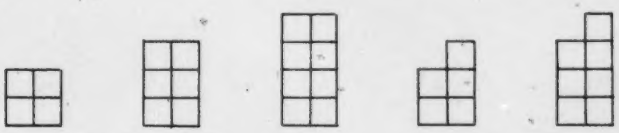
Explore other patterns in number sequences:

(odd) 1, 3, 5, 7, 9, 11, ...
(even) 22, 20, 18, 16, 14, ...
577, 578, 579, 580, ___, 582, 583, ...

Understand and use the terms *before*, *between*, and *after* for numbers 1 - 999.

Even, odd

Continue work with even and odd numbers, beginning with concrete materials and emphasizing that even-numbered sets of objects can be grouped by twos without remainders:

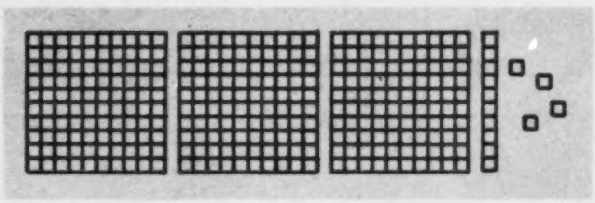


Place value, 2 and 3 digits

Continue the use of concrete models, such as metric base 10 blocks, bundles of sticks, abaci, place value charts, and base 10 rubber stamp pictures, to represent numbers beyond 10.

Examples:

314



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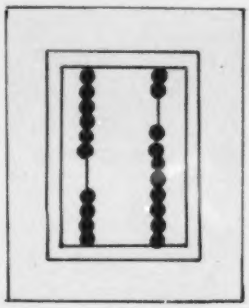


Abacus

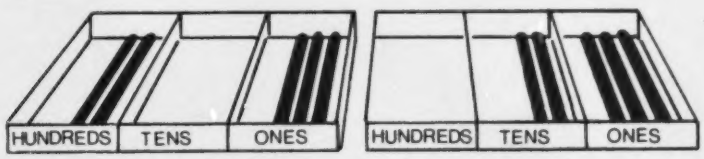
Zero

Study the meaning of zero in the place value system.

Example:

203

23



Develop an understanding of how many hundreds, tens, and ones are represented by any numeral up to 999.

Practice expanded notation for both two- and three-digit numbers.

Examples:

23 = 2 tens + 3 ones = 20 + 3

325 = 3 hundreds + 2 tens + 5 ones = 300 + 20 + 5

Fraction numerals

Develop an understanding of fractional numeration.

Money notation

Recognize dollars-and-cents notation to $10.00.

SECOND GRADE

OPERATIONS WITH WHOLE NUMBERS

Addition, subtraction through 18

Master addition and subtraction facts (sums through 18), working with concrete objects, number lines, structured materials, games, and patterns.

Equality, inequality

Continue work with inequalities using sets of concrete objects, balance scales, and the number line.

Example:

2 < 5 (2 comes before—to the left of—5 on a horizontal number line and is therefore *less than* 5)

Explore inequality in number sentences. Compare with equality.

Example:

2 + 1 < 4
2 + 1 = 3

7 - 5 > 1
7 - 5 = 2

Use the terms *greater than, less than, bigger than,* and *smaller than.*

Place value, 2 and 3 digits

Practice addition and subtraction of two- and three-digit numbers without regrouping (exchanging), using concrete objects such as metric base 10 blocks.

Study the relation between addition and subtraction facts with two- and three-digit numbers, without regrouping (exchanging).

Example:

$$\begin{array}{r} 32 = 30 + 2 \\ +24 = 20 + 4 \\ \hline 56 = 50 + 6 = 56 \end{array} \qquad \begin{array}{r} 56 = 50 + 6 \\ -24 = -20 - 4 \\ \hline 32 = 30 + 2 = 32 \end{array}$$

$$\begin{array}{r} 234 = 200 + 30 + 4 \\ +142 = 100 + 40 + 2 \\ \hline 376 = 300 + 70 + 6 = 376 \end{array} \qquad \begin{array}{r} 376 = 300 + 70 + 6 \\ -142 = -(100 + 40 + 2) \\ \hline 234 = 200 + 30 + 4 = 234 \end{array}$$

Inverse operations

Perceive that addition and subtraction are opposite *(inverse)* operations.

Regrouping

Develop the ability to add and subtract two-digit numbers requiring regrouping (exchanging), using concrete objects such as metric base 10 blocks.

Expanded notation

Practice expanded notation to add two- and three-digit numbers.

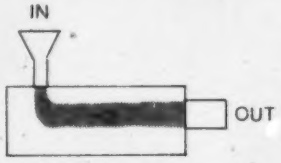
Example:

$$\begin{array}{r} 26 = 20 + 6 \\ 35 = 30 + 5 \\ \hline 50 + 11 = 50 + 10 + 1 = 60 + 1 = 61 \end{array}$$

Relations

Use tables and "machines" to show number relations.

Examples:



IN	20	19	18
OUT	17	16	15

Number sentences

Work with place holders in other than the "sum's" place in number sentences.

Examples:

$3 + \square = 14$

$\square + 2 = 12$

Patterns

Use place holders to study patterns of sums, such as "The Nine Family":

$\square + \triangle = 9$

$\square$	$\triangle$
0	9
1	8
2	7
3	6
.	.
.	.
.	.

$\square + \triangle < 9$

$\square$	$\triangle$
0	8
0	7
1	6
1	5
2	6
.	.
.	.

Multiplication

Explore multiplication facts (products) through 25, using physical objects.

Relate multiplication facts to counting by twos, threes, fours, ... on number lines and tables of numbers, 1 - 100.

Repeated addition

Relate the combination of equivalent sets to repeated addition, and to the language of multiplication.

Use arrays to picture multiplication:

3 X 2

Division

Explore division facts through 18 by relating them to multiplication facts.

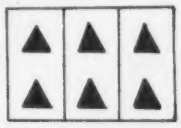
Example:

2 X 8 = 16 16 ÷ 2 = 8
8 X 2 = 16 16 ÷ 8 = 2

Sharing

Explore division as a process of sharing ("How many ... in each group?").

Example:



6 ÷ 3 = 2

Repeated subtraction

Explore division as repeated subtraction and as a process for finding the number of equivalent subsets of a given set ("How many 2's are there in 6?").

Example:

6 ÷ 2 = 3

Addition

Commutative property

Study the commutative property for addition. Does subtraction have the commutative property?

Example:

15 + 5 = 5 + 15 (yes) 15 - 5 = 5 - 15 (no)

Discover patterns the commutative property makes in tables.

Example:

+	1	2	3	4	5
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

Use the table for subtraction as well as for addition, thereby exploring the inverse relationship between the two processes.

Associative property

Discover the associative property for addition. Practice by looping paired numbers.

Example:

$(1 + 2) + 5 = 1 + (2 + 5)$

Identity

Review the fact that the sum of zero and a number is the number.

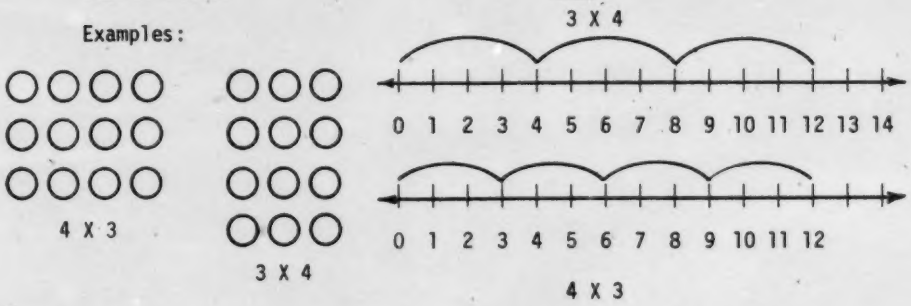
Example:

$2 + 0 = 2$
$0 + 3 = 3$

Multiplication
Commutative property

Study the commutative property for multiplication through arrays and number lines.

Examples:



Division
Inverse operations

Perceive that multiplication and division are "opposite," *inverse*, or "doing-undoing" operations.

Example:

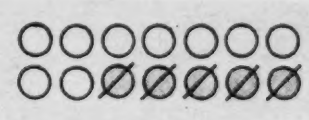
$2 \times 8 = 16$ $\quad$ $16 \div 2 = 8$
$8 \times 2 = 16$ $\quad$ $16 \div 8 = 2$

Addition, subtraction through 18
Story problems

Integrate computational skills with story problems related to daily activities, including those in other subjects. Illustrate the problems with concrete objects, pictorial representations, and/or number lines.

Example:

Scott had 14 pennies in the morning. During the day, he lost five. How many pennies did he have left?



Translate the story problem into a number sentence:

$14 - 5 = \square$

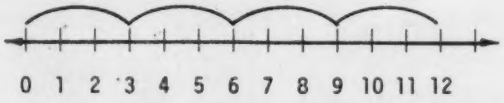
Multiplication through 25

Use story problems, pictorial representations, and/or number lines to develop multiplication skills.

Examples:

How many buttons are there in all?

Katie the Kangaroo starts at 0 and jumps three spaces at a time. Where will she be after four jumps of three?



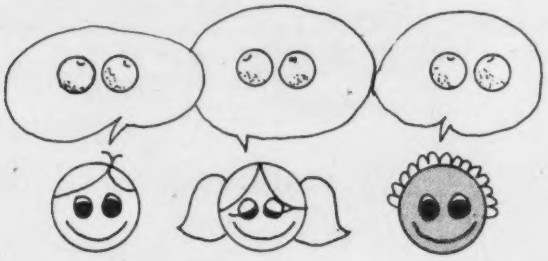
Division through 18

Use story problems, pictorial representations, and/or number lines to develop division skills.

Examples:

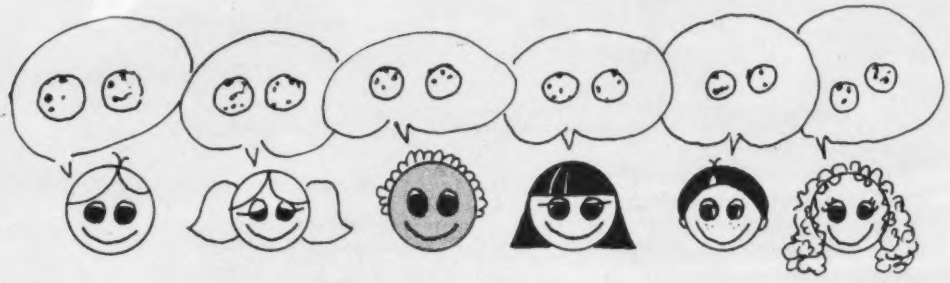
Sharing

A woman had six oranges which she shared equally among three children. How many did each child get?

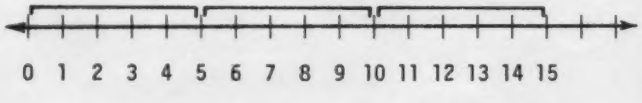


Equivalent subsets

The teacher had 12 cookies. She wished to give two cookies to each child. How many children will get cookies?



Katie the Kangaroo is at 15. How many jumps of five will she have to make to get to 0?



Closure

Deal with number sentences like $2 \div 4 = \square$, and perceive that no natural number can replace $\square$.

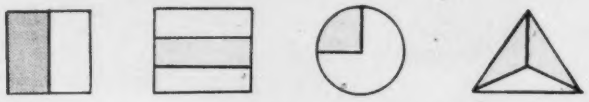
Relate fractions to use in daily life.

Equal parts of a whole

Extend the study of unit fractions from $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, to $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{100}$, using physical materials such as a meter stick.

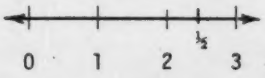
Relate the various unit fractions to one whole unit or to one group, expressing the relation as *one out of two, one out of three,* ... parts.

Shade a specified part or parts of a circle, square, rectangle, ... and name the fractional parts.



Number line

Locate halves on a number line and/or ruler.



Equal parts of a set

Find one-half, one-third, one-fourth, ... of a collection of things.

Ratio

Explore the concept of two-to-one correspondence to develop ratio thinking.

Example:

2 pieces of bread - 1 sandwich
2 legs - 1 person

Record:

Number of children	1	2	3	4	5	6
Number of eyes	2	4	6	8	10	12

Story problems
Estimation

Integrate the understanding of fractions with story problems that can be physically verified. Estimate the answers before solving the problems.

Examples:

Debbie has eight baseball cards. She gives half of them to Ron. How many cards does Debbie have left?

In a group of 12 children, one-fourth wear blue tee-shirts. How many of the children wear blue tee-shirts?

Checking

Ask whether the answers are reasonable.

Money notation

Explore simple addition and subtraction, using money notation.

Example:

$$\begin{array}{r} \$1.10 \\ +2.52 \\ \hline \$3.62 \end{array}$$

Probability

Integrate the language of fractions with the language of probability.

Data gathering

Collect and tabulate data such as body measurements (height, handspans, feet), ages, addresses, number of siblings, telephone numbers, number of times letters occur on a page, sport preferences, ...

Arrange data (temperature, attendance, ...) in tables, and illustrate them with bar graphs.

Tallying

Extend the use of tallying in games and other classroom situations.

Example:

hot lunches	𝍸 𝍸 |
cold lunches	𝍸 |||

Analysis

Compare data in terms of number, equality, and inequality (e.g., "Which was the coldest day of the month?").

Chance events

Play probability games such as tossing bean bags onto shaded sections of a polygon:

Perform experiments with three or more equally likely outcomes, using a spinner with equal sections of different colors.

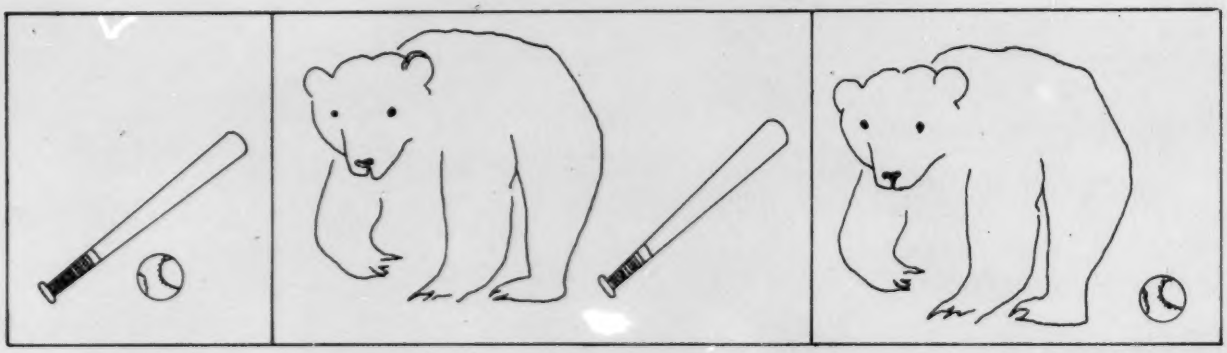
Use and discuss the language of chance, such as "one chance in two" or "one chance in three" that Henry (or Susie or Joe ...) will be in school tomorrow.

Sets

Explore making subsets of one or two things from sets of two, three, or four concrete objects:

Examples:

How many different sets of two can be made from a bat, a ball, and a bear?



How many different sets of three numbers will produce a sum of 10? Here are a few:

0 + 1 + 9 = 10	1 + 1 + 8 = 10
0 + 2 + 8 = 10	1 + 2 + 7 = 10
0 + 3 + 7 = 10	1 + 3 + 6 = 10

Arrangements	Explore making arrangements of 2 to 5 items. Example: How many "words" can be made from the letters, W, A, R, and M, when each letter is used only once? (N.B. Any arrangement of the four letters should be counted as a "word," e.g., RAMW.)

SECOND GRADE — GEOMETRY AND MEASUREMENT

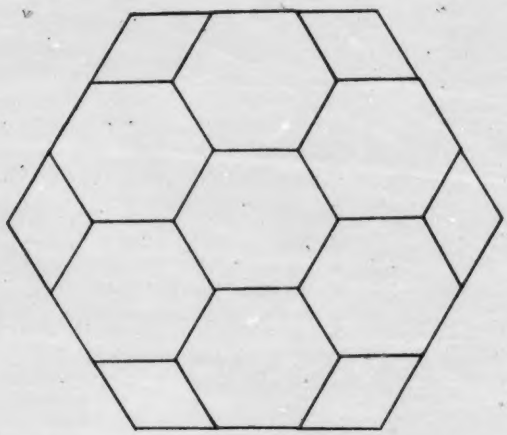
Size, measurement	Continue to explore the concepts of equality, inequality, and estimation, using actual measurements.
Nonstandard units of measure	Continue activities with nonstandard units of measure for length, distance, mass, and capacity.
Standard units of measure	Use metric units of measure. (The customary system of measure should be used only as it appears incidentally.)
Length Meter Decimeter Centimeter	Study the subdivisions on the meter stick. Use meter, decimeter, and centimeter in actual measurement situations.
Mass Kilogram Gram	Weigh various objects in grams and kilograms.
Capacity Liter Deciliter	Measure liquids and other "pourable" substances in liters and, briefly, in deciliters.
Time	Measure time in terms of days of the week, half hours, quarter hours, minutes, and seconds, using clocks and calendars.
Temperature	Make frequent readings on a Celsius thermometer.
Money	Explore various ways to make change for a dollar, including the use of quarters and half dollars.
Computation	Practice measurement problems to review basic computation skills. Use metric measures and money problems to reinforce place value concepts. Example: $\begin{array}{rrr} \$ & d & ¢ \\ 3 & 2 & 4 \\ + & 5 & 2 \\ \hline \end{array}$ $\quad$ $\begin{array}{rrr} m & dm & cm \\ 1 & 5 & 8 \\ + & 3 & 4 \\ \hline \end{array}$
Story problems	Integrate computational practice with other subject matter areas, using actual measurement tools. Example: In April, Andy's plant was 4 cm tall. In May, it was 13 cm tall. How much did it grow?
Shape 2 and 3 dimensions	Continue to observe and discuss two- and three-dimensional shapes in the environment (snowflakes, leaves, bridges, wheels, logos, ...). Study a variety of common two- or three-dimensional objects (boxes, cans, books, fruit, balls, ...) and compare such attributes as size, shape, weight, texture, and sliding or rolling properties.

Bilateral symmetry

Examine common two-dimensional shapes (sheets of paper, leaves, tee-shirts, paper plates, ...) for "folding" or "mirror" symmetry.

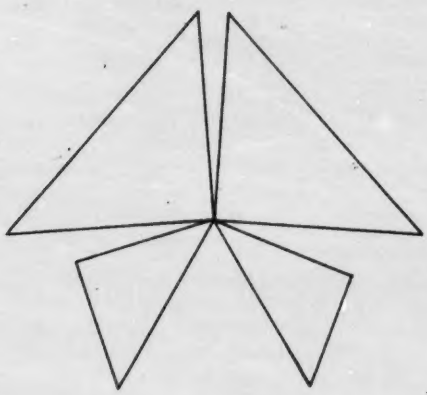
Shape fitting

Explore the component parts of space-filling geometric designs through shape puzzles, geoboards, and coloring activities.



Art

Relate geometry and design in constructive ways.



Third Grade

THIRD GRADE	NUMBER AND NUMERATION
Cardinal numbers Ordinal numbers	Use cardinal numbers through 10,000 and ordinal numbers through 500.
Counting	Skip count to 100, using all numbers from 2 through 10, on number lines and number charts.
Rounding	Experiment with rounding numbers, using a number line, a thermometer, or a meter stick.
Even, odd	Consider and discuss whether the sum (and the difference) of two whole numbers is even or odd.
Positive and negative numbers	Understand the concept of positive and negative numbers through activities involving temperatures above and below zero, elevator levels above and below ground, gains and losses in games like football, balanced and overdrawn bank accounts, ...
Large numbers Estimation	Explore the meaning of large numbers through such activities as estimating the number of grains of rice in a coffee can, the number of letters on a page, the length of a string in millimeters, the number of heartbeats in an hour, ...
Place value	Reinforce the understanding of numbers to 10,000 by using a variety of place value models.
Decimals	Use activities with money notation to explore the meaning of the places to the right of the decimal point.
Periods	Use, read, and write numerals through 10,000 (10 000), using either a comma or a space to simplify reading. (The International System recommends spaces.)
Fraction notation	Interpret nocation for fractions, including mixed numbers.

THIRD GRADE	OPERATIONS WITH WHOLE NUMBERS
Addition, subtraction	Master addition and subtraction facts to 25.
Inequality	Continue work with inequalities in number sentences. Example: From the set {1, 2, 3, 4, 5, 6, 7, 8, 9}, choose the greatest number that will make the inequality a true statement. $2 + \square < 7$
Place value, 2 and 3 digits, less than 10,000	Develop skill in adding two or more two- or three-digit numbers totaling less than 10,000, regrouping or exchanging when necessary. Example: $\begin{array}{r} 3679 \\ +\ 2875 \\ \hline 6554 \end{array}$

Regrouping (Exchanging)

Continue to use manipulative aids such as base 10 blocks or abaci to reinforce the concept of regrouping (exchanging).

Develop skill in finding the difference between two numbers, each less than 10,000, using regrouping when necessary.

Metric measure, money notation

Use metric measures and money notation to reinforce the understanding of place value.

Expanded notation

Practice expanded notation to reinforce the understanding of place value.

Multiplication tables

Develop skill and quick numerical responses to multiplication facts through 100, using audiovisual materials such as blocks, arrays, grids, and number lines.

Develop the following table and perceive the pattern caused by the commutative property:

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2											
3											
4											
5											
6											
7											
8											
9											
10											

Study the algorithm* for multiplication of two- and three-digit multiplicands by one-digit multipliers. (Use an intermediate algorithm briefly for understanding.)

Example:

Intermediate Algorithm

```
  627          627
   X8           X8
   56         5016
  160
 4800
 5016
```

Division

Develop skill and quick numerical responses to division facts under 100, relating them to multiplication.

Use common division terminology.

Example:

Relate "How many times does 7 go into 56?" to "What does 56 *divided by* 7 equal?"

*See *Algorithms*, p. 2.

2 and 3 digits

Study short and long algorithms for division, using intermediate algorithms briefly.

Example:

Intermediate Algorithm

$$\begin{array}{r} 3 \\ 30 \\ 7\overline{)231} \\ 210 \\ \hline 21 \\ 21 \\ \hline \end{array} \qquad \begin{array}{r} 33 \\ 7\overline{)231} \\ 21 \\ \hline 21 \\ 21 \\ \hline \end{array} \qquad \begin{array}{r} 33 \\ 7\overline{)231} \end{array}$$

Symbols

Explore various ways a division problem can be presented.

Example:

$3\overline{)19}$ $\qquad 19 \div 3 = \square$ $\qquad \frac{19}{3} = \square$

Develop and solve story problems, using division.

Example:

Gary wants to distribute 36 marbles equally among six boxes. How many marbles should he put into each box?

Properties

Reinforce and develop the understanding of commutative and associative properties, identity, and inverse operations, by applying them to the solution of problems.

Example:

Two students copied three numbers from the chalkboard for addition. Laurie wrote 18 + 7 + 24. Julie wrote 7 + 24 + 18. How many different ways are there of writing the problem? What is the sum in each case?

Multiplication Identity

Note the fact that the product of any number times 1 is the number:

$\square \times 1 = \square$

Zero

Note the fact that the *product* of any number *multiplied* by zero is zero. The *sum* of any number *added* to zero is the number.

$\square \times 0 = 0$ $\qquad \square + 0 = \square$

Commutative property

Study the commutative property in sentences and multiplication tables. Discover the symmetrical relationship in the multiplication 100-chart:

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Division

Inverse operations

Continue to study factor and product relationships, showing multiplication and division as opposite (inverse) operations.

Examples:

$22 \times 11 = 242$

$242 \div 22 = 11$

Story problems

Addition, subtraction

Extend whole number addition and subtraction techniques to story problems in other subject areas and to money problems, using decimal notation.

Example:

Lisa bought art supplies that cost $7.36. She gave the salesman a ten-dollar bill. How much change did she get?

Multiplication

Solve basic multiplication story problems with two- and three-digit numbers multiplied by one-digit numbers.

Example:

An experienced mechanic earns four times as much as his/her helper. If the helper earns $5.00 an hour, how much does the mechanic earn an hour?

Division

Solve basic division story problems with two- and three-digit numbers divided by one-digit numbers.

Example:

David has 288 stamps in his collection. He puts nine stamps on a page. How many pages of stamps are there in the collection?

Use multiplication (inverse operation) to check the answers to division story problems.

Introduce calculators to reinforce base 10 notation as well as to check the answers to problems involving large numbers.

Games, patterns, integration

Reinforce skills in operations with whole numbers by doing puzzles, games, pattern discoveries, calculator exercises, and cross-disciplinary projects in art, crafts, social studies, ...

Inequalities

Multiplication and addition

Consider inequality statements involving multiplication and division.

Example:

Identify four whole numbers that can be put into the frame in the following inequality.

$\square \times 50 < 210$

THIRD GRADE

OPERATIONS WITH FRACTIONS

Practical uses

Identify the use of fractions in daily life: a half-hour TV program, an eighth note in music, a quarter pound of butter, ...

Order, inequality

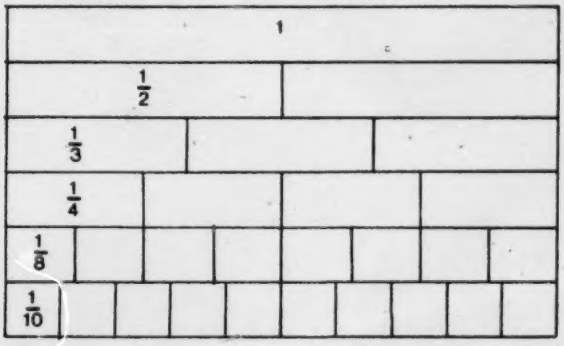
Study the concept of order with commonly used fractions, including such terms as *greater than, less than,* and *between.*

Example:

$\frac{1}{2} > \frac{1}{3} > \frac{1}{4}$ $\quad \frac{1}{6} < \frac{1}{5} < \frac{1}{4}$

Equivalence

Develop an understanding that two halves, three thirds, four fourths, ... are equal to one whole. Use concrete models, number lines, and pictures.



Equal parts of a whole

Review the meaning of unit fractions, including $\frac{1}{8}$.

Note that a diameter of a circle and a diagonal of a rectangle or a square cut the respective figures into halves.

Equal parts of a set

Develop the meaning of such fractions as $\frac{2}{3}$ and $\frac{3}{4}$, for both wholes and collections of things. Use the terms *numerator* and *denominator.*

Estimation

Estimate answers before computation. For example, "The sum of $\frac{1}{4}$ and $\frac{1}{5}$ is less than $\frac{2}{4}$."

Ratio

Develop the concept of ratio in practical problem-solving situations.

Examples:

3 cans of corn ⟶ 99¢
120 kilometers ⟶ 2 hours
1 centimeter ⟶ 10 meters

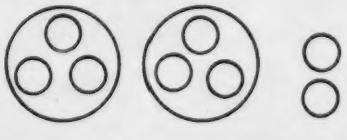
Relation to division

Develop an understanding of the relationship between fractions and division.

Example:

$8 \div 3$ may be written $\frac{8}{3}$

$\frac{8}{3} = \frac{6}{3} + \frac{2}{3} = 2 + \frac{2}{3} = 2\frac{2}{3}$



$8 \div 3 = 2\frac{2}{3}$

Measurements

Take measurements to the nearest tenth of a meter.

Read halves and fourths of inches and tenths of centimeters on rulers and meter sticks.

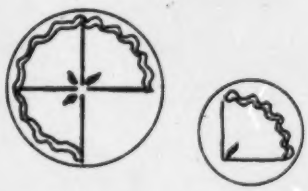
Money notation

Continue to use decimal notation to represent dollars and cents.

Addition, subtraction

Add and subtract fractions having like denominators, using concrete objects or pictorial representations.

Examples:



$$\frac{4}{4} - \frac{1}{4} = \frac{\square}{4}$$

Story problems

Continue to integrate fractions with story problems which can be concretely verified.

Example:

Kathy has three marbles. One marble is red and two are blue. The blue marbles represent what fractional part of the set of marbles?

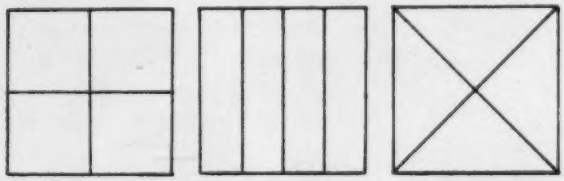
Decimals

Add and subtract decimal fractions to tenths.

Equal parts

Explore such problems as the following:

How can a square be divided into four equal parts?



THIRD GRADE

PROBABILITY AND STATISTICS

Data gathering

Continue to collect data (results from sports contests, popularity polls for movies, car makes, comics, ...), and organize them in tables and bar graphs.

Collect and discuss graphs found in newspapers and in magazines.

Chance events
Estimation of outcomes

Estimate (predict) the outcomes of experiments, do the experiments, and then compare the results with the predictions.

Example:

Estimate the number of heads that will turn up if you flip a coin 100 times. Then flip the coin and compare the results with the prediction.

Fractions

Develop story problems and experiment with chance happenings (probabilities), using fractions to express the results.

Example:

A bag has three apples and one pear. If Julie picks a fruit without looking, what are her chances of picking an apple?

Arrangements

Find an orderly way of counting the number of possible arrangements within a set, using concrete objects.

Example:

Arrange three colored chips in a line in as many different ways as possible.

Record the arrangements, using the following code:

R = red, W = white, B = blue

RWB RBW BRW BWR WBR WRB

Unequally likely events

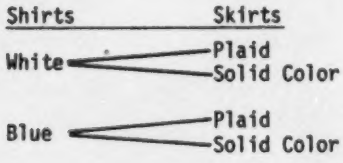
Investigate activities with unequally likely outcomes such as the spinner game illustrated below.

Tree diagrams

Use *tree diagrams* to study possible combinations.

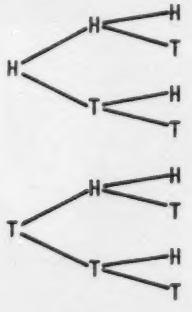
Example:

How many different outfits can Mary wear if she has one white shirt, one blue shirt, one plaid skirt, and one solid color skirt?



Example:

How many different outcomes are possible if a coin is tossed three times?



List arrangements	Arrange two chairs one behind the other. Then choose two classmates and discuss the number of different ways they could be assigned to the chairs. Perform the experiment, recording the data on the chalkboard or a newsprint pad. Repeat the activity, using three chairs and three people. Example: Alice Alice Frank Frank Mary Mary Frank Mary Alice Mary Alice Frank Mary Frank Mary Alice Frank Alice

THIRD GRADE — GEOMETRY AND MEASUREMENT

Measurement Tools	Use metric measuring tools in activities involved with length, mass, capacity, and temperature.
Equivalence	Identify equivalent measures within a system of measure. Examples: 3 m = 300 cm 2 kg = 2,000 g 4 L = 40 dL 1 yd. = 36 in. $5.00 = 500 ¢
Time	Study time to five- and one-minute intervals. Relate the clock to circle construction and to fractions. Example: The tip of the hour hand sweeps through what part of a circle in one hour?
Money	Develop and solve problems created from menus, supermarket flyers, and mail-order catalogs. Study money to $10.00, emphasizing decimal notation.
Volume	Explore the use of three-dimensional shapes (boxes, jars, cans, ...) to "fill" space.
Perimeter	Find the perimeters of polygons, using rulers.
Story problems	Develop and solve measurement problems related to other subject areas, using measurement tools. Coordinate the development of fraction concepts (fourths, eighths, tenths, hundredths, ...) with the use of measurement tools.
Shape	
Construction	Study shapes by working with actual construction, measurement, and space-visualization problems.
Plane	Study the properties of plane figures (circles, quadrilaterals, pentagons, hexagons, ...), using geoboards, straws, toothpicks, ... Note points, line segments, and angles, including right angles or "square corners."
Compass and protractor	Use compasses and protractors to construct circles.

Solids

Study the properties of solid figures (spheres, cubes, pyramids, ...), noting vertices, line segments, edges, and angles.

Example:

How many edges, faces, and vertices are there on a brick?

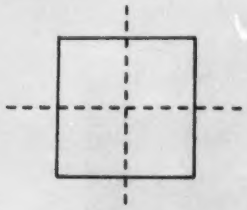
Continue to explore the use of geometric shapes to "fill" space in a plane.

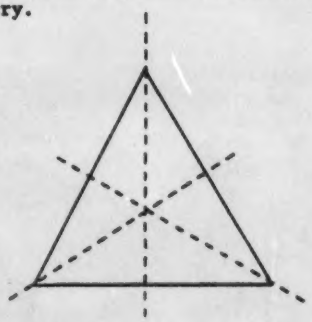
Example:

Lines of symmetry

Continue symmetry experiments with plane figures. Discover figures with more than one line of symmetry.

Example:



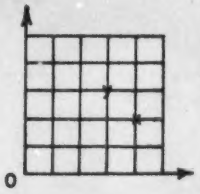


Coordinates
Position

Locate positions on a grid.

Example:

Place an x four steps to the right of zero and two steps up.
Place a y three steps to the right of zero and three steps up.

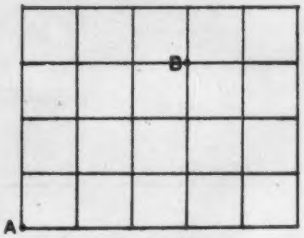


Locate points (4,3) and (3,3) on a grid. The ordered pair of numbers (4,3) identifies a point four units to the right of (0,0) and three units up.

Give directions for getting from one place to another on a map, using horizontal and vertical paths.

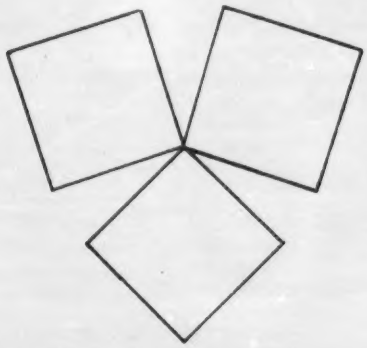
Example:

Describe a path from A to B.



Geometric design

Continue using geometric figures to make pictures and designs.



Fourth Grade

FOURTH GRADE — **NUMBER AND NUMERATION**

Cardinal numbers

Read and write whole numbers to hundred millions.

Counting

Skip count to numbers greater than 100.

Rounding

Practice rounding numbers to the nearest tenth, whole number, hundred, and thousand, using devices such as number lines.

Learn the convention for rounding numbers like 15 to the nearest ten (in this case, the answer is 20).

Even, odd

Study the *evenness* or *oddness* of the sum, the difference, and the product of two numbers.

Prime

Examine prime numbers less than 100. (Reminder: A prime number is a natural number greater than one, the only factors of which are one and itself.) Explore primes on charts such as Eratosthenes' Sieve.

Integers

Continue activities and discussions involving the concept of positive and negative numbers, with applications in daily life. Extend the use of the number line and coordinates to include negative numbers.

Large numbers, estimation

Explore quantitative information which will help to relate personal experiences to the meaning of *million*.

Place value

Extend the concept of place value to millions and hundredths.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	tenths	hundredths
7,	8	0	4,	9	6	1 .	3	7
millions			thousands			ones		

Roman system

Compare Arabic and Roman systems of numeration (from I through C) with respect to place value and basic symbols.

Patterns

Perceive patterns in sequences of numbers.

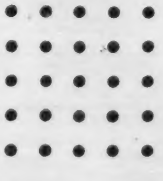
Examples:

Triangular Numbers

1, 3, 6, 10, 15

Square Numbers

1, 4, 9, 16, 25



Addition, subtraction

Practice computing sums and differences of whole numbers, using numbers less than one million.

Safeguard routines
Estimation

Use the *estimate, compute, and check* routine in computational problems.

Estimate sums and differences prior to actual computation.

Examples:

654	700	600
397	400	300
+238	+300	+200
	1400	1100

The sum is less than 1400 but greater than 1100.

963
- 89

The difference is less than 900 but greater than 800.

Checking

Use the inverse relationship between addition and subtraction to check computation.

Example:

735 - 548 = 187 because 187 + 548 = 735.

Missing addends

Solve number sentences like the following:

735 - N = 187

Renaming, subtraction

Solve subtraction problems involving renaming with zeros.

Example:

5000
-1423
3577

Factors

Identify prime factors.

Example:

24 has the factors 1, 2, 3, 4, 6, 8, 12, and 24.

Prime factors

2 and 3 are prime factors.

Practice prime factorization of composite numbers less than 50.

Example:

24 = 2 X 2 X 2 X 3

Common factors

Find the common factors of several numbers less than 100.

Multiples

Find the greatest common factors and the least common multiples of sets of whole numbers.

Examples:

G.C.F. (21, 27, 30) = 3 L.C.M. (2, 7, 8) = 56

Multiplication facts through 144

Develop quick numerical responses to multiplication facts with products to 100. Extend to facts with products through 144.

2 and 3 digits

Use the standard multiplication algorithm for three-digit numbers multiplied by two-digit numbers. Use the intermediate algorithm, if necessary, for clarification.

$$\begin{array}{r} 427 \\ \times\ 36 \\ \hline 2562 \\ 1281 \\ \hline 15372 \end{array}$$

Intermediate Algorithm

$$\begin{array}{r} 400 + 20 + 7 \\ \times\quad 30 + 6 \\ \hline 42 \\ 120 \\ 2400 \\ 210 \\ 600 \\ 12000 \\ \hline 15372 \end{array}$$

by 10

Practice written and mental multiplication first by 10 and 100, then by 1000 and 10,000.

Division

Continue to use the number line and arrays to illustrate the relationship between multiplication and division facts.

through 144

Review division facts with dividends to 100. Extend to facts with dividends to 144.

Example:

$8 \times 11 = 88$
$88 \div 8 = 11$

2 and 3 digits

Find the quotient and remainder in division problems using a one-digit divisor and a three-digit dividend.

Extend the division process to division by two-digit divisors.

Checking

Check multiplication and division examples by using the inverse relationship between the two processes.

Example:

$420 \div 7 = 60$ because $60 \times 7 = 420$

Use concrete models to show the inverse relationship between multiplication and division involving fractions.

Example:

$\frac{48}{8}$ is equivalent to $\frac{1}{8} \times 48$.

Properties
Commutative
Associative

Review examples which illustrate the commutative and associative properties of multiplication.

Distributive

Use the distributive property of multiplication over addition or subtraction in problem solving.

Example:

$7 \times 23 = 7 (20 + 3) =$

$(7 \times 20) + (7 \times 3) =$

$140 + 21 = 161$

Study the related distributive property of division over addition or subtraction.

Example:

$$50 \div 8 = \frac{48 + 2}{8} = \frac{48}{8} + \frac{2}{8} = 6\frac{2}{8}$$

Identity

Review the concept of zero as the identity element for addition.

Recognize the number 1 as the identity element not only in multiplication, but also in division.

Inverse operations

Review multiplication and division as inverse (opposite) operations, using number lines and arrays.

Word problems

Use diagrams and open sentences to help solve verbal problems.

Multiple-step

Begin to solve *multiple-step* problems.

Example:

> An airplane traveled from New York to St. Louis by way of Chicago. There were 183 passengers when it left New York. In Chicago, 75 passengers got off and 86 got on. How many passengers were on the plane when it left Chicago?

Multiplication

Solve word problems which involve multiplication.

Example:

> The lighthouse signal blinks once every 5 seconds. How many times does it blink in 23 minutes?

Division

Solve word problems which involve division.

Example:

> Mrs. Lopez rented eight dozen chairs for a meeting. How many rows of chairs can she make if she puts eight chairs in each row?

Ratio

Solve word problems, using the concept of ratio intuitively.

Example:

> To get the right color, Jan mixes two liters of white paint with three liters of red paint. How many liters of red should she mix with six liters of white in order to get the right color?

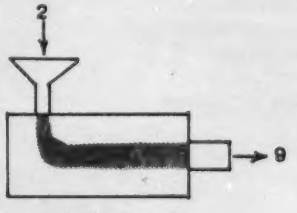
Pattern relations

Continue to explore number patterns and relations.

Examples:

Multiply by 5 and subtract 1.

1	2	3
4	9	14



Search for a rule and fill in the missing numbers in the following table:

1	2	3	4	5	6
1	3	5		9	

Integration activities

Reinforce the understanding of operations with whole numbers through appropriate games, puzzles, and cross-disciplinary projects and activities.

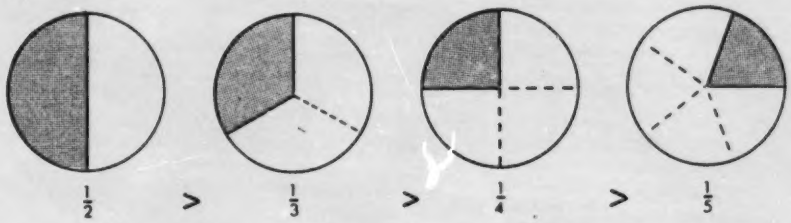
FOURTH GRADE

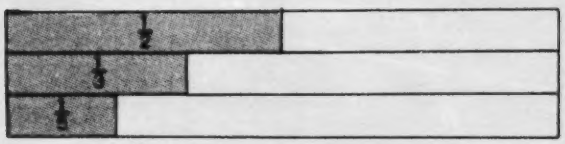
OPERATIONS WITH FRACTIONS

Order
Inequality

Study the order of unit fractions.

Examples:



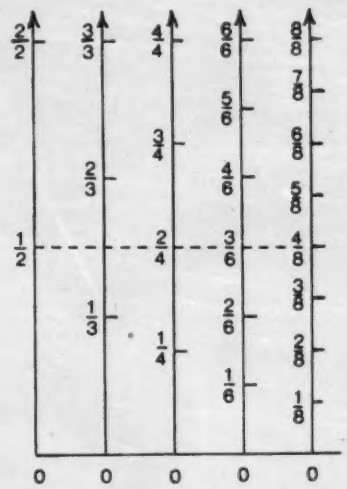


$\frac{1}{5} < \frac{1}{3} < \frac{1}{2}$

Locate points for numbers such as $2\frac{1}{2}$, $3\frac{1}{4}$, and $6\frac{3}{4}$ on a number line.

Equivalence

Compare fractions on a coordinated set of number lines:



Write numerals for a set of fractions equivalent to any given fraction.

Example:

Write three fractions equivalent to $\frac{1}{3}$.

Equal parts of a unit or a set

Review fractions as representing equal parts of a unit or a set. Use concrete and/or visual materials, when needed.

Prime factoring

Consider fractions with numerators and denominators factored into primes.

Example:

$$\frac{6}{10} = \frac{3 \times 2}{5 \times 2}$$

Addition, subtraction

Common denominators

Add and subtract fractions with common denominators.

Example:

$$\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$$

Unlike denominators

Add and subtract fractions with unlike denominators. (Briefly)

Multiplication rule

Example:

$$\frac{1}{2} + \frac{1}{3} \qquad \frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \qquad \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

Decimals

Observe the decimal notation when reading masses on food packages.

Use number lines and base 10 blocks to illustrate fractions, including decimals.

Practice using decimal notation for tenths and hundredths, and correlate common-fraction notation for tenths and hundredths with decimal notation.

Explore the order relationship of common fractions and decimal fractions, using equivalent fractions with common denominators to arrange fractions in order:

$$\frac{1}{2} < \frac{2}{3} < \frac{3}{4} \text{ or } \frac{6}{12} < \frac{8}{12} < \frac{9}{12}$$

$$0.3 < 0.36 \text{ or } 0.30 < 0.36$$

Explore decimal fractions to thousandths, as applied to millimeters.

Develop the skills of addition and subtraction of decimal fractions to hundredths.

Relate fractions to the monetary system in the United States, and to metric measurements.

Examples:

1 dollar = 100¢ = \$1.00

$\frac{1}{2}$ dollar = 50¢ = \$ .50

0.5 km = 500 m

0.5 kg = 500 g

6 cm = 0.6 dm

Word problems

Solve problems that illustrate the presence of fractions, including decimals, in the environment.

Example:

Debbie lives 0.7 km from school. Tommy lives 0.3 km from school. How much closer to school does Tommy live?

Ratio

Continue to develop the concept of ratio in everyday problem-solving situations.

Examples:

5 candy bars	$1.00
4 cm	100 km
1000 m	10 minutes

Multiplication Decimals

Multiply decimal fractions (to tenths).

Example:

$$\begin{array}{r} 2.1 \\ \times\ 3.6 \\ \hline 126 \\ 63 \\ \hline 7.56 \end{array}$$

Practice locating decimal points in products by estimation.

Example:

3 X 2.6 is approximately 3 X 3. Since

3 X 3 = 9, 3 X 2.6 must be less than 9.

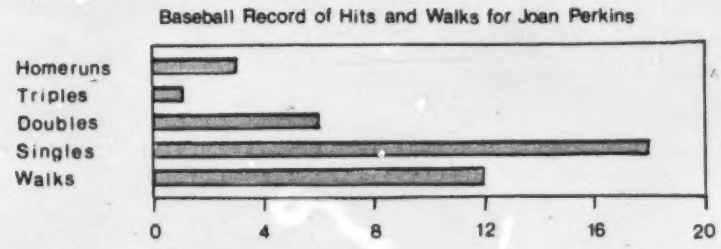
FOURTH GRADE

PROBABILITY AND STATISTICS

Data gathering and organization

Make graphic records of statistical data drawn from newspapers, magazines, polls, appropriate activities in other subject areas, ...

Example:



Frequency tables

Make frequency tables from tallied data.

Example:

The frequency with which given letters occur in a poem, a story, or on a page.

Letter	*Tally*	*Frequency*
A	1111	4
B	11	2
C	~~1111~~	5

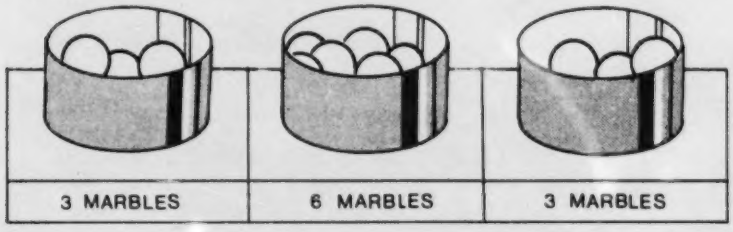
Analysis

Average or arithmetic mean

Use concrete materials to develop the concept of *average* or *arithmetic mean*.

Examples:

Rearrange the marbles so that each cup has the same number.



What is the number?

Experiment with two dice. Roll them five times. What is the average of the five sums?

Range

Examine the *range* (the difference between the largest and the smallest numbers in a set) and the *mean* (average) in a collection of data, while organizing the information in the form of graphs or tables.

Chance events

Prediction

Continue to predict outcomes of actual probability experiments.

Fractions

Continue to study probabilities in terms of fractions by participating in actual experiments.

Example:

A hat contains six folded pieces of paper, each with a different one of the following numbers written on it: 1, 2, 3, 4, 5, and 6. What are Billy's (Susie's, Tommy's, ...) chances of picking an even-numbered slip?

Unbiased sample

Explore informally the conditions that must be checked in order to achieve *an unbiased random sample* (i.e., a set in which every member has an equal chance of being chosen) in data gathering, and its practical use in television ratings, public opinion polls, and marketing surveys.

Arrangements

Explore problems that involve a systematic identification of ordered arrangements using models, pictures, lists, or tree diagrams for organizing data.

Example:

In how many ways can the doctor schedule appointments for eye examinations for David, Jessica, and May?

DJM	JDM	MJD
DMJ	JMD	MDJ

FOURTH GRADE — GEOMETRY AND MEASUREMENT

Measurement

Tools

Continue to use appropriate metric measurement tools (including tape) in measurement activities which involve length, distance, mass, capacity, and temperature.

Equivalence

Continue to compare equivalent measures within the metric system.

Units of measure

Identify appropriate units for measuring the area, mass, perimeter, volume, ... of a variety of common objects.

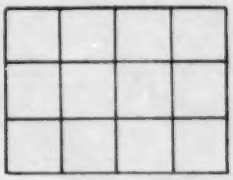
Example:

How can a block be measured (length, mass, volume, ...), and what are the appropriate units for each form of measure?

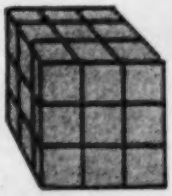
Area, volume, perimeter

Find the area, volume, and perimeter of specific figures, initially by counting units.

Examples:



Area = 12 square units



Volume = 27 cubic units

Circumference

Use coordinate paper and string to estimate the areas and circumferences of circles.

Formula

In the study of area and volume, extend the counting of units into work with factors and multiplication.

Time

Study time to the second.

Word problems

Develop and solve measurement problems related to topics in other subject areas.

Examples:

Science

A satellite in orbit travels 480 km each minute. How many km per hour does it travel?

Social Studies

Plan a trip through a foreign country. Identify the places you'd like to visit and then use a map to determine the distance between these places (in km, of course!), the amount of petrol (gasoline) needed to go from one place to another in the car or cycle of your choice (measured in liters), the population density per square kilometer, ...

Health

Weigh everyone in the class and tabulate and graph the data in kilograms.

Shape

Construction tools

Use straight-edge rulers, protractors, and compasses to construct geometric figures informally, in two dimensions.

Plane and solid figures

Continue to explore common plane and solid geometric figures in the environment, as well as in objects and drawings.

Terms

Learn to use such terms as *polygon, chord, radius, face, edge, vertex, angle, line segment,* and *point* while working with problems.

Develop an intuitive understanding of such terms as *parallel*, *perpendicular*, and *intersecting*.

Example:

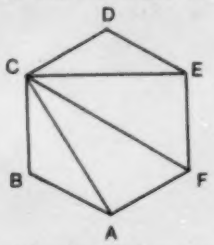
Use a geoboard to find line segments *parallel* or *perpendicular* to given line segments.

Identify *parallelograms* in the environment.

Identify the diagonals of polygons.

Example:

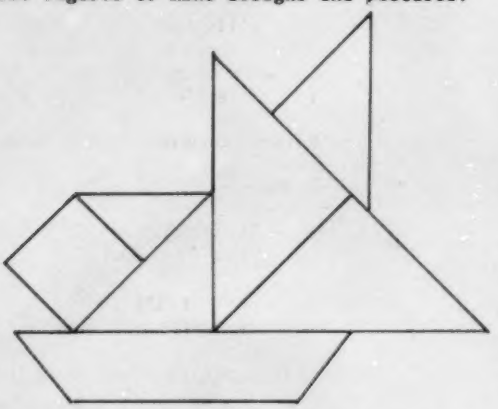
How many diagonals can be drawn from a vertex (see C, below) of a polygon?



Designs

Use geometric figures to make designs and pictures.

Example:



Symmetry

Examine the *symmetry* or patterning in tables like the following:

x	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Coordinates

Extend work in coordinate geometry to both positive and negative coordinates.

Example:

Use line segments to connect the following points in order:

(0,2), (2,2), (4,$1\frac{1}{2}$), (6,3), (5,1), (6,-1), (4,0),

(2,-1), (-3,-2), (-5,-2), (-6,$-1\frac{1}{2}$), (-3,-1),

(-6,-1), (-6,0), (-5,1), (-3,2), (0,2).

Fifth Grade

FIFTH GRADE — NUMBER AND NUMERATION

Cardinal numbers

Read and write numerals for whole numbers to one billion, remembering that numerals for large numbers have "periods" of hundreds, tens, and ones.

Example:

hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
2	5	7	8	0	4	9	6	1
millions			thousands			ones		

Rounding

Continue rounding numbers to the nearest 10 000, and extend the process to hundredths.

Estimation

Relate rounding skills to estimation.

Examples:

The product of 63 X 48 is greater than 60 X 40 and less than 70 X 50.

What is the smallest (largest) three-digit number that can be rounded to 900?

Integers

Relate positive and negative numbers to real situations and games.

Example:

Mary owed Dan $5.00 Then she borrowed $3.00 more. How much does she owe him all together?

Continue using the number line as an aid to understanding negative numbers.

Renaming through exponents

Using powers of 10, develop an understanding of *exponents* as another way of naming numbers, especially large numbers.

Examples:

$10^1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 10 \times 10 \times 10 = 1000$
$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$
$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000$
$10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10{,}000{,}000$
$10^8 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000{,}000$
$10^9 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000{,}000$

Place value

Use exponential notation to reinforce place value concepts.

Example:

$$\begin{aligned} 1492 &= 1000 + 400 + 90 + 2 \\ &= (1 \times 1000) + (4 \times 100) + (9 \times 10) + 2 \\ &= (1 \times 10^3) + (4 \times 10^2) + (9 \times 10^1) + 2 \end{aligned}$$

Understand the different meanings of a digit in numerals such as 6 in the following:

607
4567
96

Decimals

Continue to develop an understanding of the place value concept and notation with decimal fractions to thousandths, using place value models.

Example:

6.327 is a shorthand way of expressing

$$6 + \frac{3}{10} + \frac{2}{100} + \frac{7}{1000} \text{ or } 6\,\frac{327}{1000}$$

Roman numerals

Briefly consider Roman numerals through MM.

Numeration

Reinforce the concept that a number can be symbolized by many different numerals.

Example:

$$\frac{3}{4} = \frac{6}{8} = \frac{75}{100} = 0.75$$

FIFTH GRADE

OPERATIONS WITH WHOLE NUMBERS

Addition, subtraction, multiplication, and division

Continue to develop skills with addition, subtraction, multiplication, and division algorithms, extending the work to three-digit multipliers and three-digit divisors.

Ensure quick recall and mastery of basic addition, subtraction, multiplication, and division facts.

Safeguard routines

Continue the *estimate-compute-check* routine in all computational practice.

Explore and use different ways of checking and verifying computation (e.g., hand-held calculators).

Order of operations

Learn to use the conventional rule for order of operations. In expressions such as the one below, the indicated operations should be performed in the following order:

(1) All exponentiations, first;
(2) All multiplications and divisions, next; and
(3) All additions and subtractions, last.

If an expression also contains parentheses, the same order of operations must be used *within all sets of parentheses* before being applied to the expression as a whole.

Example:

$4^2 + 6 \div 2 - 3 \times 5$

First do the exponentiation.

$\underline{16} + 6 \div 2 - 3 \times 5$

Next, do all multiplications and divisions.

$16 + \underline{3} - \underline{15}$

Finally, do all additions and subtractions.

$\underline{4}$

Thus

$4^2 + 6 \div 2 - 3 \times 5 = 16 + 6 \div 2 - 3 \times 5 = 16 + 3 - 15 = 4$

Greatest common factor
Least common multiple

Continue to compute the greatest common factors and the least common multiples in sets of whole numbers.

Properties

Continue to use the commutative, associative, and distributive properties in actual computational practice and problem solving.

Continue to use inverse (opposite) operations as well as inverse and identity numbers.

Word problems, integration

Apply basic computation skills to problems from other subject areas.

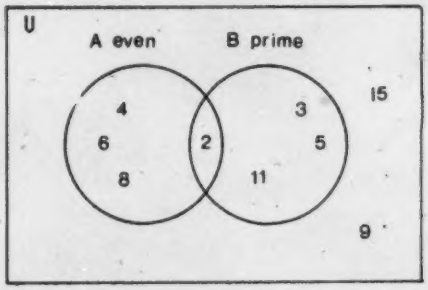
Example:

> On July 20, 1969, Buzz Aldrin and Neil Armstrong landed on the moon. Astronaut Armstrong was 38 years old. What was the year of his birth?

Sets, terms, and notation

Study basic set ideas and terms, such as *universal set, subset, equivalent sets, intersection, union,* and *complement,* using Venn diagrams and set notation.

Examples:

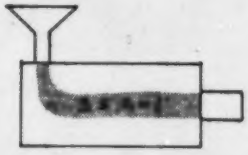


Set U = 2, 3, 4, 5, 6, 8, 9, 11, 15
Set A "even" = 2, 4, 6, 8
Set B "prime" = 2, 3, 5, 11
Set C, complement of A = 3, 5, 11, 9, 15
Set D, intersection of A and B = 2
Set E, union of A and B = 2, 4, 6, 8, 3, 5, 11

Patterns
Relations

Continue to develop computation skills and the ability to recognize patterns.

Example:



Input	1	2	3	4	5
Output	2	5	8	11	14

Integration with other subject areas

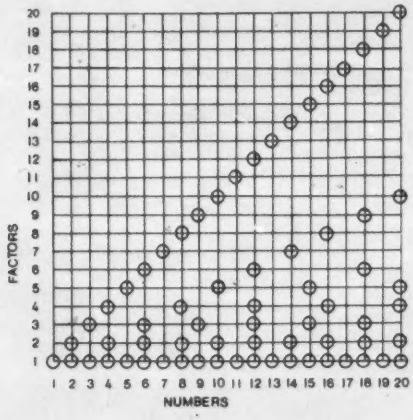
Continue to use computation skills in investigation studies in other subject areas, and in games.

Graphing

Correlate graphing with computation skills.

Example:

Make a *factor* graph.



Observe that 1 and the number itself are always factors of a number, and that common factors and prime numbers can be identified.

FIFTH GRADE — OPERATIONS WITH FRACTIONS

Review previous work with fractions.

Order, inequality

Reinforce the concepts of *greater than*, *less than*, and *betweenness* with regard to fractions.

Example:

Arrange a set of fractions $\left[\frac{3}{4}, \frac{1}{5}, \frac{2}{3}, \frac{1}{2}, \frac{1}{4}\right]$ in order from smallest to largest.

Equivalence

Practice writing equivalent fractions, including decimals.

Reducing fractions

Determine the simplest form of a fraction. (N.B. In lowest terms, the numerator and the denominator of a fraction have no common factors except 1.)

Proper, improper

Develop the concept that all *proper* fractions name numbers that are greater than 0 and less than 1.

Develop skill in changing improper fractions to equivalent *mixed* numbers, and the reverse.

Example:

$$\frac{15}{8} = \frac{8}{8} + \frac{7}{8} = 1 + \frac{7}{8} = 1\frac{7}{8} \qquad 2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$$

Factoring

Use factoring techniques to determine common denominators.

Addition, subtraction

Continue the addition and subtraction of fractions with common and unlike denominators.

Multiplication, division

Use pictures and/or other graphic illustrations to demonstrate multiplication and division of fractions.

Examples:

Multiplication $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Division

The number of thirds in 3 is $3 \div \frac{1}{3}$ or 9.

Decimals

Continue to correlate decimal and common fractions, using number lines and block models.

Rounding

Practice rounding decimal fractions.

Addition, subtraction

Review addition and subtraction of numbers with tenths and hundredths in decimal form. Extend to thousandths.

Multiplication, division

Extend multiplication and division (using whole number divisors) to hundredths.

Examples:

$$\begin{array}{r} 2.50 \\ \times \quad 3 \\ \hline \end{array} \qquad 5 \overline{)10.50}$$

Extend decimal concepts by dividing numbers by powers of 10, including the placement of the decimal point when a number is multiplied or divided by a power of 10.

Identity

Use the multiplicative identity number (1) to obtain equivalent decimal fractions, and apply this skill to the division algorithm.

Examples:

$\frac{9.0}{0.3} \times \frac{10}{10} = \frac{90}{3}$; $0.3 \overline{)9.0} = 3 \overline{)90}$

$0.9 = 0.90 = 0.900 \ldots$

Word problems

Solve problems that illustrate the use of fractions (including decimals) in everyday life.

Examples:

- Butter costs $2.80 per kg. How much money is needed to pay for 3 kg of butter?
- A person needs about 2.5 L of water each day to survive. For how many days could a person survive on 50 L of water?
- A printer whose hourly wage is $6.00 earns "time and a half" for working overtime (more than 8 hours a day). How much does he earn for working $11\frac{1}{2}$ hours on Tuesday?

Data gathering
Organization

Continue to explore many different ways of collecting and organizing data, such as tallying and using tables, graphs, and diagrams.

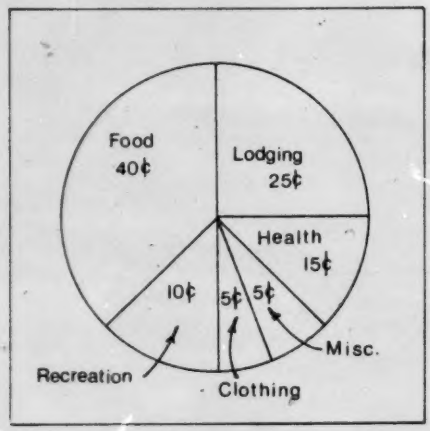
Compare bar, line, picture, and circle graphs which present the same information, and note the advantages and disadvantages of each form.

Circle graph

Use compasses and protractors to prepare circle graphs.

Example:

HOW I SPEND MY DOLLARS



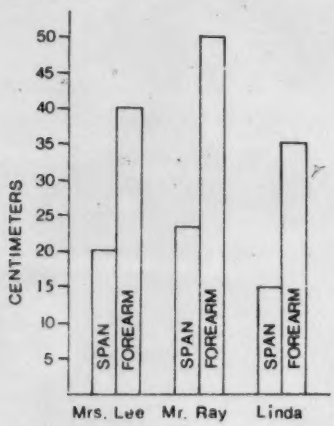
Double bar graph

Develop double bar graph skills.

Example:

Measure handspans and forearms of other children and some adults in the school. Round the measurements to the nearest cm. Then draw a double bar graph to compare the measurements.

HANDSPANS AND FOREARM MEASURES IN OUR SCHOOL



Analysis
Range
Mean

Analyze collected data (e.g., the highest or lowest temperature of each day during a given month) according to *range* and *mean* (average).

Chance events
Prediction

Continue to make predictions based on sample data and to verify the predictions with further experiments or additional research.

Example:

Identify the last digit in 25 telephone numbers drawn at random from class members or the telephone book. Are all digits (0 through 9) equally likely to appear?

Fraction

Continue to develop the idea that the chance or probability of an event's happening can be expressed as a fraction:

$$\text{The estimated probability of an event's happening} = \frac{\text{The number of times the event happens}}{\text{The total number of experiments}}$$

Reliability

Continue to conduct probability experiments, using an increasing number of trials so that the results become more reliable. For example, spinning a coin 20 times may render a probability estimate of 0.40 that heads will turn up. Spinning the coin 100 times will render a probability estimate closer to 0.50.

Impossibility

Identify events that are *impossible* (that have a chance or probability of happening equal to zero).

Example:

What is the probability that a yellow marble will be drawn from a bag containing two red marbles and three green marbles?

$$P(\text{yellow}) = \frac{\text{The number of yellow marbles}}{\text{The number of marbles in the bag}} = \frac{0}{5} = 0$$

Certainty

Identify events that are *certain* (that have a chance or probability of happening equal to one).

Example:

What is the probability that the sum of the measures of two sides of a triangle is greater than the measure of the third side?

$$\frac{\text{The number of triangles in which the sum of two sides is greater than the third side}}{\text{The number of triangles investigated}} = \frac{N}{N} = 1$$

Sometimes

Explore events that occur *sometimes*, with their chance of happening expressed as a proper fraction.

Example:

What is the probability that a blue marble will be drawn from a bag containing one red, one white, and one blue marble?

$$P(\text{blue}) = \frac{\text{The number of blue marbles}}{\text{The number of marbles in the bag}} = \frac{1}{3}$$

Random samples

Experiment with *random* and unbiased *samples* (those in which every member of the group has an equal chance of being chosen).

Example:

Examine a random and unbiased sample of the books in the school library to determine — without doing an actual count — how many of the volumes are at least 20 years old.

Sample space

Represent and count the number of elements in the *sample space* (the listing of all the possible outcomes) of an experiment.

Example:

Toss a nickel and a dime. List the possible outcomes.

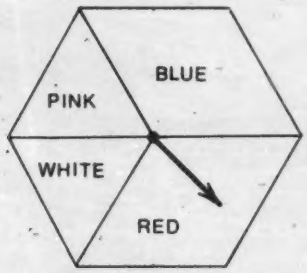
NICKEL	Heads	Heads	Tails	Tails
DIME	Tails	Heads	Tails	Heads

Integration, geometry

Use compasses to construct polygons for spinner experiments that relate probability to ratios of areas.

Example:

What is the probability that the spinner will stop on red?



Unequally likely

Investigate experiments that have outcomes which are *not* equally likely.

Example:

Select two whole numbers at random. What is the probability that their product is an even number?

Fractions
Sets which total 1

Relate sets of fractions which total 1: $\left\{\frac{1}{3}, \frac{2}{3}\right\}$, $\left\{\frac{2}{5}, \frac{3}{5}\right\}$,... to sets, sample spaces, and probability.

Examples:

If a die is tossed, what is the probability that the number that comes up will be less than 6?

$$P(6) = \frac{1}{6}$$

$$P(< 6) = 1 - \frac{1}{6} = \frac{5}{6}$$

If a card is picked from a regular deck of cards, what is the probability that it will *not* be an ace?

$$P(\text{ace}) = \frac{4}{52}$$

$$P(\text{not an ace}) = 1 - \frac{4}{52} = \frac{48}{52}$$

The probability of winning a game is $\frac{2}{7}$. What is the probability of *not* winning?

$$P(\text{win}) = \frac{2}{7} \qquad P(\text{not win}) = 1 - \frac{2}{7} = \frac{5}{7}$$

Tables and graphs

Continue to collect statistical data and record it in tables and/or graphs which clearly indicate change.

Example:

Keep a daily (weekly, monthly) record of home runs, running or swimming speeds, test scores, temperatures, stock market prices, ... in ways that will indicate the differences (increase and decrease) between entries.

THE ROLLING SKATEBOARD COMPANY

DATE	*CLOSING PRICE*	*CHANGE*
Jan. 15	$87\frac{1}{2}$	
Feb. 15	$72\frac{3}{8}$	$-15\frac{1}{8}$
Mar. 15	$81\frac{1}{4}$	$+8\frac{7}{8}$
Apr. 15	$68\frac{7}{8}$	$-12\frac{3}{8}$

FIFTH GRADE

GEOMETRY AND MEASUREMENT

Measurement

Continue to measure length, distance, mass, volume, capacity, ... with metric tools and measures. Be thoroughly familiar with the prefixes *milli-*, *centi-*, and *kilo-* and with such symbols as *g*, *mg*, *kg*, *mL*, *L*, *mm*, *km*, and *cm*.

Area
Perimeter

Continue to work with area and perimeter, using centimeter graph paper and/or centimeter blocks.

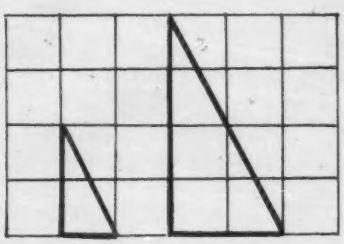
Example:

Outline your hand (or someone else's hand) on graph paper and estimate the length of the line (perimeter) and the size of the space (area) defined by it.

Develop formulas for the area and perimeter of rectangles and squares.

Measure the area and perimeter of triangles, circles, irregular polygons, ..., using a concrete discovery approach and cut paper, geoboards, lattices, and centimeter blocks or graph paper.

Example:



Volume
Capacity

Continue to measure volume and capacity, using cubic centimeter blocks and metric measuring tools.

Temperature

Continue to measure temperatures (indoor; outdoor; of substances like milk, soup, sand, salt, boiling water; ...) with a Celsius thermometer.

Practice computation

Develop skills in operations with metric units.

Graphs

Use graph skills in measurement projects related to weather.

Shape

Plane figures

Continue to draw and measure plane geometric figures, using rulers, compasses, and protractors.

Similarity
Symmetry
Congruence

Use concrete and artistic activities to explore the concepts of *similarity*, *symmetry*, and *congruence* in plane geometric figures.

Scale
Ratio

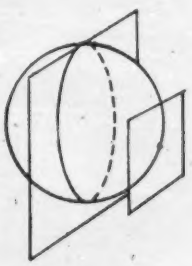
Use centimeter grids to explore the concept of scale drawing, and relate *scale* to *ratio*.

Solid

Practice identifying the intersection of planes and three-dimensional geometric figures.

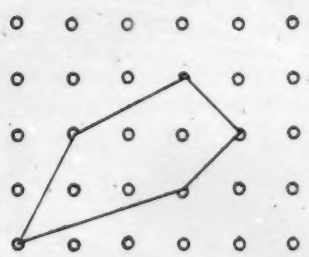
Example:

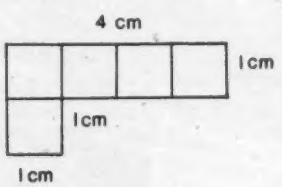
The intersection of a plane and a sphere is a circle or a point.



Explore area measurement with triangles, circles, and irregular polygons, using a concrete discovery approach on a geoboard, a lattice, or centimeter graph paper.

Examples:





Coordinates

Continue graphing exercises with ordered pairs of numbers.

Sixth Grade

SIXTH GRADE — NUMBER AND NUMERATION

Number

Review reading, writing, and understanding large numbers.

Rounding

Continue to round numbers (including decimals), particularly while making estimates in computation.

Negative numbers

Extend the use of negative number notation to fractions.

Exponents

Use the exponential form of powers of 2, 3, 5, and 10. Relate these forms to factoring.

Example:

$5^3 = 5 \times 5 \times 5 = 125$

$2^2 \times 3^3 = (2 \times 2) \times (3 \times 3 \times 3) = 108$

Examine briefly the use of exponents in scientific notation.

Example:

$6.25 \times 10^2 = 625$

$7.5 \times 10^4 = 75{,}000$

Ratios

Use colons (:) to indicate ratios.

Example:

1:4 means *a ratio of one to four*, which is often expressed $\frac{1}{4}$.

SIXTH GRADE — OPERATIONS WITH WHOLE NUMBERS

Addition, subtraction, multiplication, and division

Continue to improve skills with addition, subtraction, multiplication, and division algorithms.

Factoring

Continue practice with greatest common factors and lowest common multiples.

Integers

Intuitively consider the use of integers in relevant activities (loss and gain in bank transactions, rushing records in football, temperature changes, ...).

-5 degrees Celsius

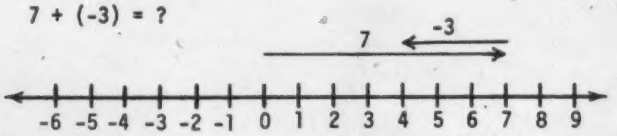
Example:

At 6 a.m., the temperature was -5°C. By noon, it was 9°C. How many degrees warmer was it at noon than at at 6 a.m.?

Consider the addition of integers as steps on a number line.

Example:

$7 + (-3) = ?$



$7 + (-3) = 4$

Properties

Review the use of basic laws of arithmetic (commutative, associative, and distributive) as applied to operations and algorithms.

Inverse operations

Be aware of inverse operations, addition/subtraction and multiplication/division.

Example:

Understand that the product of a number and its multiplicative inverse (reciprocal) is 1,

$3 \times \frac{1}{3} = 1$

and the sum of a number and its additive inverse is 0.

$3 + (-3) = 0$

Open sentences

Write and solve open sentences dealing with inverse operations, using letters as well as frames as place holders.

Example:

What numbers make the following statements true?

$3 \times n = 36$ $\qquad$ $\square + 36 = 84$

$n = 36 \div 3$ $\qquad$ $\square = 84 - 36$

Word problems

Write and solve open sentences while working with word problems.

Example:

The area of a rectangle is 84 square centimeters. One side of the rectangle is 12 cm long. Find the length of the other side.

$12 \times W = 84$

$W = 84 \div 12$

Sets and set notation

Continue the use of set notation and Venn diagrams to illustrate the union and intersection of sets in real situations (sports teams, committees, attendance, ...).

SIXTH GRADE

OPERATIONS WITH FRACTIONS

Fractions

Review operational skills with common and decimal fractions and mixed numbers.

Common
Decimal

Example:

$3\frac{2}{10} + \frac{1}{4} = \square$ $\qquad$ $3.2 + 0.25 = \square$

Estimation

Practice estimating the result of an operation before computing it.

Examples:

- $\frac{3}{4} + \frac{5}{6} + \frac{2}{3}$

  The sum is greater than 2, but less than 3.

- $4\frac{1}{3} - 2\frac{5}{6}$

  The difference is greater than 1, but less than 2.

- $\frac{2}{3} \times \frac{4}{5}$

  The product is less than 1.

- $\frac{5}{8} \div \frac{3}{4}$

  The quotient is less than 1.

Inequality

Continue to explore the concepts of *greater than*, *less than*, and *betweenness* for fractions.

Equivalence

Use the *cross product* rule for comparing fractions:
If the cross products of two fractions are equal, then the fractions are equal.
If the cross products of two fractions are unequal, then the fractions are unequal.

Examples:

- Does $\frac{3}{4} = \frac{9}{12}$?

  $\frac{3}{4} \not< \frac{9}{12}$ (3 X 12 and 4 X 9 are the cross products)

  3 X 12 = 4 X 9, therefore $\frac{3}{4} = \frac{9}{12}$

- Does $\frac{2}{3} = \frac{5}{7}$?

  2 X 7 ≠ 3 X 5, therefore $\frac{2}{3} \neq \frac{5}{7}$

Common denominator method

Practice the *common denominator* method in order to determine the equivalence of fractions.

Example:

$$\frac{2}{5} \boxed{?} \frac{4}{9}$$

$$\frac{2}{5} = \frac{2}{5} \times \frac{9}{9} = \frac{18}{45} \qquad \frac{4}{9} = \frac{4}{9} \times \frac{5}{5} = \frac{20}{45}$$

$$\frac{18}{45} < \frac{20}{45}$$

Therefore $\frac{2}{5} < \frac{4}{9}$

Decimal fractions

Practice using decimal fractions — with or without a hand-held calculator — to compare common fractions.

Example:

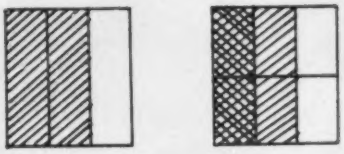
$\frac{3}{7} \square \frac{2}{5}$ $\qquad 0.429 > 0.400 \qquad$ $\frac{3}{7} > \frac{2}{5}$

Multiplication Algorithm

Discover and use the algorithm for multiplying fractions, going back to a pictorial approach when necessary.

Example:

$$\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} \text{ or } \frac{1}{3}$$



N.B. Some teachers introduce multiplication of fractions before introducing addition of fractions.

Practice multiplying a variety of combinations of proper and improper fractions and whole and mixed numbers.

Example:

$$3 \times 2\frac{1}{2} = \frac{3}{1} \times \frac{5}{2} = \frac{3 \times 5}{1 \times 2} = \frac{15}{2} = 7\frac{1}{2}$$

Commutative property

Discover the commutative property for the multiplication of fractions.

Example:

$$\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2} = \frac{2}{6} \text{ or } \frac{1}{3} \qquad \frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} \text{ or } \frac{1}{3}$$

Distributive property

Use the distributive property in the multiplication of mixed numbers.

Example:

$$\begin{aligned} 3 \times 2\frac{1}{2} &= 3 \times (2 + \frac{1}{2}) \\ &= (3 \times 2) + (3 \times \frac{1}{2}) \\ &= 6 + \frac{3}{2} \\ &= 6 + 1\frac{1}{2} \\ &= 7\frac{1}{2} \end{aligned}$$

Word problems

Use the basic properties and the multiplication algorithm with real-life illustrations.

Example:

Water covers $\frac{5}{7}$ of the earth's surface; $\frac{19}{20}$ of this is salt water. What fraction of the earth's surface is salt water?

Division

Understand that division by a number is equivalent to multiplication by the reciprocal of the number.

Example:

$$3 \div 4 = 3 \times \frac{1}{4}$$

Make the division of fractions meaningful by using a variety of approaches (*factor-product, partitioning, grouping, common denominator, reciprocal, ...*).

Decimals

Continue to develop skills with decimal fractions, using the four basic operations.

Rounding

Continue rounding decimals to the nearest tenth or hundredth.

Inequality

Explore inequality of decimal fractions.

Example:

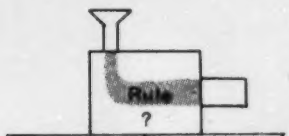
Arrange 1.2, 0.004, 0.02, 0.003, and 0.1 so that the numbers are in order from smallest to largest.

Relations

Extend skills with relations to fractions, including decimals.

Example:

Fill in the missing numbers in the table below:



input	output
2	5
5	11
8	17
1/2	?
?	31
2.5	?

Percent

Understand the concept of percent through problems related to actual situations, 100-squares, and number lines.

Example:

There are 100 students in the sixth grade. Of these, 78 can swim. What percent of the class can swim?

$$\frac{78}{100} = 78\%$$

As a common fraction

Convert common fractions and decimal fractions to percent.

Examples:

$$\frac{3}{100} = 3\% \qquad \frac{100}{100} = 100\ \% \qquad \frac{1}{2} = \frac{50}{100} = 50\%$$

$$0.06 = 6\% \qquad 0.60 = 60\% \qquad \frac{5}{2} = 2\,\frac{1}{2} = 2.50 = 250\%$$

As ratio

Develop the idea of percent as a *ratio* (a fraction with a denominator of 100).

Example:

In a set of five marbles, three are blue. What percent of the marbles are blue?

To solve the problem, find a fraction which:
(a) has a denominator of 100, and (b) is equivalent to $\frac{3}{5}$.

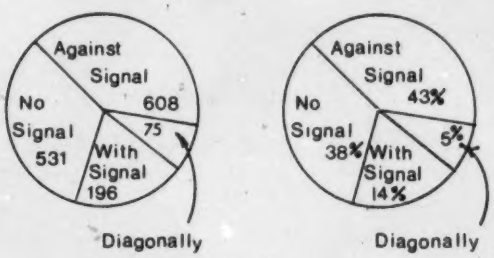
$\frac{3}{5} = \frac{\square}{100}$ $\quad \frac{3}{5} = \frac{60}{100}$ Therefore 3 is 60% of 5 (i.e., 60% of the marbles are blue).

Circle graphs

Use circle graphs to explore the concept of percent.

Example:

TRAFFIC INJURIES TO CHILDREN CROSSING AT INTERSECTIONS



Ratio

Extend the use of fractional notation to ratio and probability.

Examples:

- $\frac{3}{5}$ means a ratio of 3 to 5 or 3:5
- $\frac{3}{5}$ may mean 3 chances out of 5.

Proportion

Understand that an equality between two ratios is called a *proportion*, and that the cross products of a proportion are equal.

Example:

$\frac{5}{6} = \frac{10}{12}$ or 5:6 = 10:12

$\frac{5}{6} \times \frac{10}{12}$ thus 5 X 12 = 6 X 10

Find the missing number in a proportion in which three of the numbers are known, and letters are used as place holders.

Example:

$\frac{2}{3} = \frac{n}{12}$

3 X n = 2 X 12

3 X n = 24

n = 8

Word problems

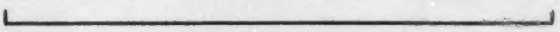
Use ratios and proportions in developing and solving real-life problems.

Examples:

Mrs. Stein's car travels 28 km in 20 minutes. How many km can Mrs. Stein travel in an hour?

$\frac{28}{20} = \frac{n}{60}$

The scale on a map indicates that one centimeter has been used to represent approximately ten kilometers. If the distance between Albany and Kingston equals this line segment

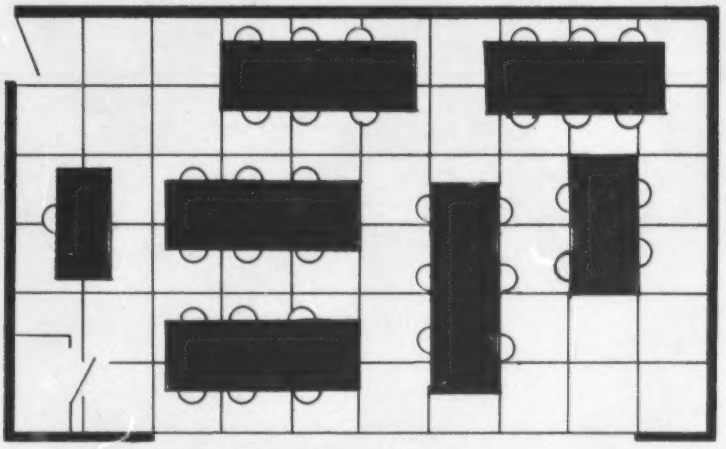


on the map, what is the distance between the two cities in kilometers?

Scale drawing

Make a scale drawing of the classroom (your bedroom, a "dream" room, a store, ...).

Example:



SIXTH GRADE

PROBABILITY AND STATISTICS

Data gathering and organization

Continue to collect, organize, analyze, and illustrate data compiled from opinion polls, experiments, reference books, ...

Participate in extended recordkeeping projects.

Example:

Keep a record of the highest and lowest daily temperatures. What conclusions can be drawn and what predictions can be made from the data?

Analysis
Central tendency

Consider uses and misuses of averages in interpreting data.

Example:

The salaries of four people are (1) $15,000; (2) $20,000; (3) $22,000; and (4) $125,000. Find the average salary. Discuss.

Chance events

Investigate the use of probability and statistics in careers.

Definition/equation

Apply the definition of probability to problems.

$P(E) = \frac{f}{n}$ ↔ The *probability* of an *event* = $\frac{\text{The number of } \textit{favorable} \text{ outcomes}}{\text{The total number of } \textit{possible} \text{ outcomes}}$

Example:

What is the probability that a card picked from a regular deck of cards will be the eight of diamonds?

$P(8 \diamondsuit) = \frac{1}{52}$

Independent events

Perform probability experiments with *independent* events (those in which one event has no influence on another).

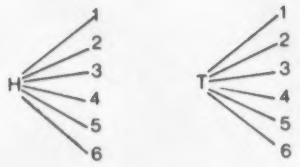
Example:

What is the probability that the head side will turn up on a coin toss and the 6 on a die toss?

$P(\text{Head}) = \frac{1}{2}$

$P(6) = \frac{1}{6}$

POSSIBILITIES	
H, 1	T, 1
H, 2	T, 2
H, 3	T, 3
H, 4	T, 4
H, 5	T, 5
H, 6	T, 6



$P(\text{Head and } 6) = \frac{1}{12}$

Unbiased samples

Discuss the importance of unbiased sampling while studying such real-life issues as energy consumption, pollution, taxes, inflation, ...

Discuss *un*representative samples.

Sample space

Continue to make diagrams and lists of sample spaces.

Example:

What is the probability that a three-child family includes two boys and one girl?

sample space:

B B B	G G G
B B G	G G B
B G B	G B G
B G G	G B B

$P(\text{2 boys and 1 girl}) = \frac{3}{8}$

Arrangements
Permutations

Practice counting the number of possible arrangements (*permutations*) that can be made with a set of objects, when different orders are considered different arrangements.

Examples:

How many "words" can be made from the letters A, B, and C?

ABC	BAC	CAB
ACB	BCA	CBA

How many three-digit numbers can be made using digits 0 - 9?

100	200	. . . . .	900
101	201	. . . . .	901
102	202	. . . . .	902
103	203	. . . . .	903
104	204	. . . . .	904
105	205	. . . . .	905
106	206	. . . . .	906
107	207	. . . . .	907
108	208	. . . . .	908
109	209	. . . . .	909

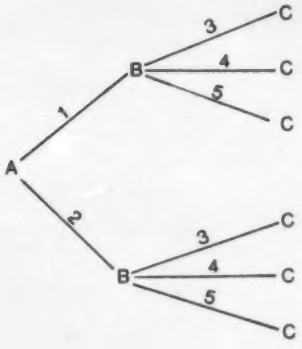
Multiplication principle

Discover the *multiplication principle* through experiences with tree diagrams or rosters (lists) of possible events taken in order.

> Multiplication Principle: If one task can be done in M ways and another task can be done in N ways, there are M times N ways to do the two tasks.

Example:

> From Alhambra City (A) to Bolivar City (B), there are two different roads. From Bolivar City to Crinkle City (C), there are three different roads. How many different routes are there from Alhambra City to Crinkle City?



Possible Routes

Road	Road
1	3
1	4
1	5
2	3
2	4
2	5

> There are 2 x 3 or 6 different routes from Alhambra City to Crinkle City.

Combinations

Practice counting the number of groups of a given size that can be made from a set of objects when different orders are *not* considered as different arrangements.

Example:

> How many different groups of two children can be made from a group of five children: Abby, Bonnie, Carlos, Dennis, and Ellie?
>
> AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

SIXTH GRADE — GEOMETRY AND MEASUREMENT

Measurement

Continue metric measurement activities designed to increase familiarity with units of length, distance, capacity, and mass.

Practice general computation skills in all operations with a variety of metric measures.

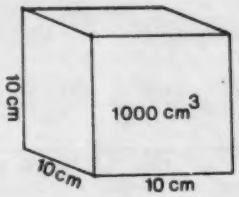
Area
Perimeter

Maintain skills in finding the areas and perimeters of plane figures.

Volume

Continue to work with volume in practical activities as well as in word problems. Explore the volume of cylinders empirically.

Relate volume to capacity in terms of metric measure (cubic centimeters, kilograms, liters).



Metric/customary system of measure

Use approximations when relating metric units to customary units of measure.

Examples:

When using the metric system, there is seldom any necessity for conversion to customary units. However, the following approximations may be useful in drawing comparisons between the two systems:

A yard is a little less than a meter (about 10% less).
A foot is about 30 cm.
An inch is about 2.5 cm.
A quart is a little less than a liter.
A pound is a little less than half a kilogram.
A mile is about 1.6 km.

Shape

Continue to do construction activities (duplicating segments, bisecting angles, ...) with compasses, protractors, rulers, ...

Plane

Use compasses for constructing regular hexagons, Venn diagrams, circle graphs, and similar designs.

Angles

Identify acute, obtuse, and right angles.

Measure angles with a protractor.

Similarity

Investigate intuitively the concept of similarity in a variety of polygons.

Symmetry
Congruence

Continue investigations of symmetry and congruence, using curve stitching, drawings, and shapes in the environment.

Scale

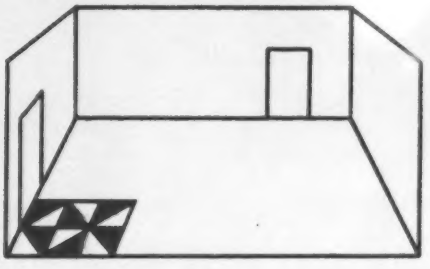
Practice scale drawing by making floor plans and similar geometric designs on graph paper.

Integration

Solve word problems with geometric design and measurement settings.

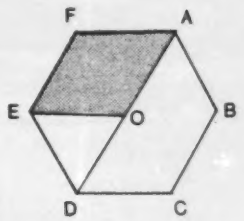
Example:

How many 22-cm square tiles will be needed to cover the floor of a rectangular room 3 meters long and 2 meters wide?



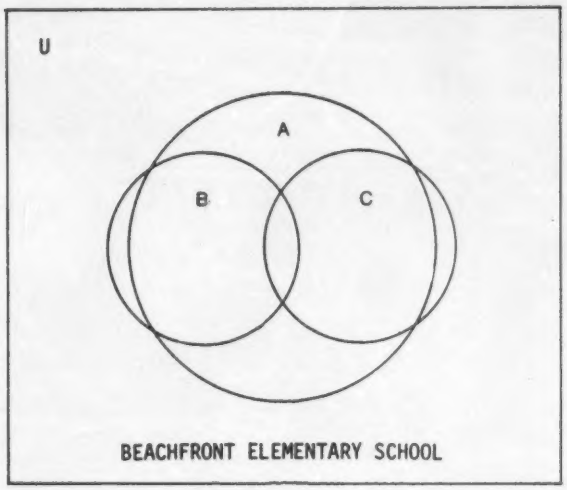
Use a discovery approach to integrate the study of fractions and ratio with the study of shape and area.

Examples:

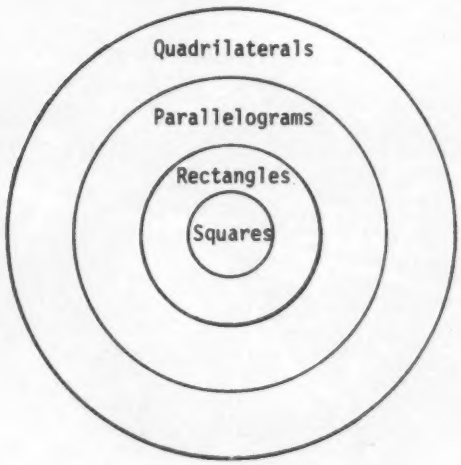


The area of the trapezoid A B C D is equal to half the area of the hexagon shown above, and the area of the rhombus A O E F is equal to a third of the area of the hexagon. What is the area of the triangle O D E relative to the area of the hexagon?

Use circle diagrams to illustrate set classifications.



A = 6th graders
B = volleyball team
C = softball team
U = total number of students



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